Stress Dependency of Rock Mass Modulus In Predicting Closure of Underground Openings

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ABSTRACT

A correct understanding of the deformability characteristics of rock mass is essential to estimate the closure of underground openings subjected to a given stress field. The rock mass modulus is generally obtained from classification techniques or field tests, and is treated stress-independent in the analysis. Evidences are available in literature, which indicate that the rock mass modulus varies with in situ stresses. This paper presents results from a laboratory study which was conducted to highlight the effect of stress dependency of rock mass modulus in prediction of closure of underground openings in nearly elastic ground conditions. Specimens of jointed rock mass were prepared with D-shaped model opening excavated in the mass. Different joint configurations and in situ stress conditions were used. Observations on closure of opening and internal support pressure were taken. The closure for a given set of in situ stress conditions and internal support pressures was computed using the stress-independent modulus. These predictions were found to be different from laboratory test results. The observed ground reaction curves are non-linear. To improve the predictions, stress-dependent rock mass modulus was used and satisfactory results were obtained. Based on this study, an empirical expression has been suggested to take into account the stress dependency of rock mass modulus in the prediction closure of underground openings.

Keywords: Underground openings; Closure; Ground response curve; Pressure dependency; Rock mass modulus

1. INTRODUCTION

Rock masses in the field are generally in equilibrium in presence of far-field in situ stresses. An excavation for underground opening disturbs this equilibrium and causes re-distribution of stresses around the opening. The induced stresses thus produced cause closure of the opening. If an internal support system is installed it interacts with the deforming rock mass and provides a check on the further deformation by developing support pressure through a complex mechanism involving stiffnesses of the support and the surrounding rock mass. Assessment of internal support pressure corresponding to a given closure of the opening is, therefore, an integral part of any design process of underground openings. Closed form solutions are available for predicting the closure of a circular opening in an isotropic elastic rock mass (Bray, 1987). The most important parameter governing the displacements is the rock mass modulus. Generally the rock mass modulus is treated as a constant in the linear elastic theory. However there are evidences that rock mass modulus depends on the stress (Lionco and Assis, 2000; Kulhawy, 1975; Santarelli et al., 1986; Brown et al., 1989). The assumption of stress independent rock mass modulus may, therefore, introduce appreciable error in predicting the closure of an underground opening. Review of the present literature does not indicate any comprehensive experimental study where the effect of stress dependency of rock mass modulus on closure of the underground openings has been studied. This paper highlights results from an experimental study, in which the closure and the internal support pressure of a model opening in a jointed rock mass were observed in the laboratory. The closures were then predicted by using an elastic theory and compared with the laboratory test results to study the effect of stress dependency of rock mass modulus on the closure of the opening. Expressions have also been suggested to consider the stress dependent rock mass modulus in the analysis. The study assumes non-squeezing elastic isotropic ground conditions.

2. EXPERIMENTAL PROGRAMME

Very few laboratory studies have been reported on closure of underground openings in jointed rock masses. The reason seems to be the fact that these studies involve large number of elemental rock blocks which are difficult to handle. Further, simulation of in situ stresses in the rock mass and monitoring of closure of opening for an applied internal pressure is a difficult problem. It was decided to face these challenges through a systematically planned and executed experimental study. Details of such a comprehensive study are available in Choudhari (2007). The study involved the use of pre-cast elemental rock blocks to simulate the specimens of jointed rock mass. The rock mass specimens contained two sets of joints and a D-shaped model opening supported by rigid supports. To simulate far field in situ stresses, the rock mass specimen was subjected to a bi-axial stress field. In field generally plane strain conditions prevail. However, in this study plane stress condition was generated due to experimental difficulties. The excavation of the opening was simulated by allowing an incremental closure of the opening by displacing the supports. Variations in the support pressure due to closure of the opening were monitored. The experimental programme involved the following steps:

i) Design and fabrication of a large size reaction frame

A reaction frame, capable of withstanding 500 kN force both in horizontal and vertical directions, and having a clear internal spacing of 1500 mm x1500 mm was designed and fabricated (Fig.1). Hydraulic jacks were used to apply the loads in horizontal and vertical directions and hence generate in situ stresses in the rock mass around the opening. The deformation of the rock mass was monitored during the application of in situ stresses using LVDTs in both horizontal and vertical directions to get the rock mass modulus.

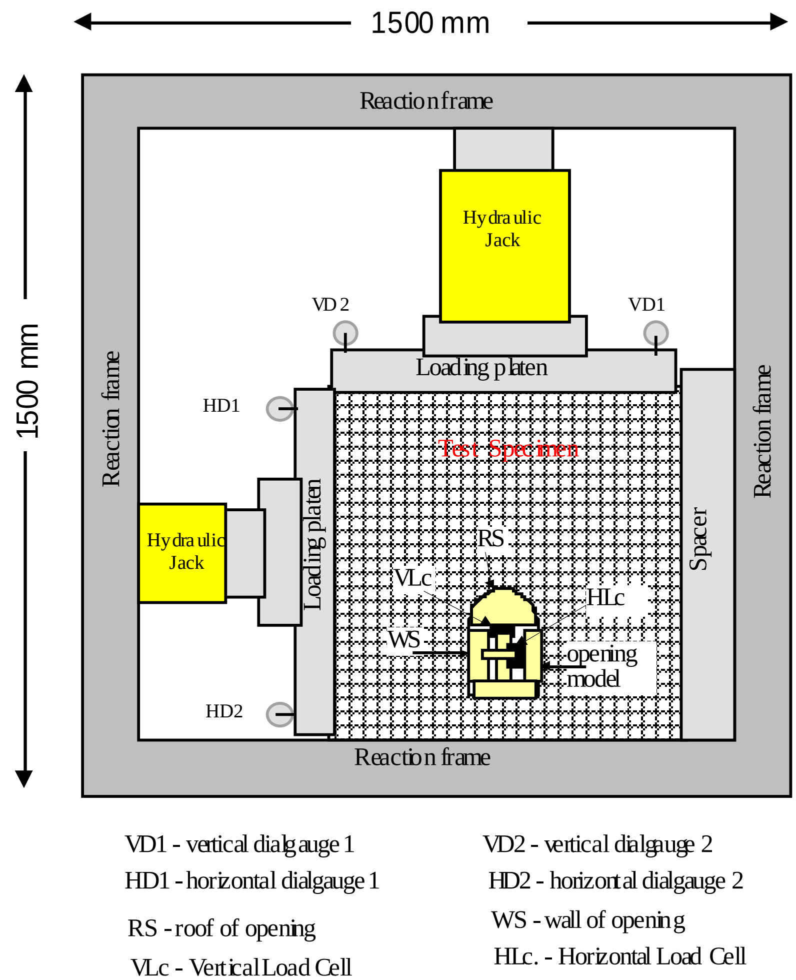

Fig.1 - Reaction frame with model of opening in rock mass

ii) Selection and characterization of a suitable model material

A large number of rock blocks (around 1800) were required to form the model rock mass specimen. Rock mass was formed by using blocks of Plaster of Paris which is quite common in experimental studies. The composition of material comprised of plaster of Paris, sand and water mixed in a ratio 1:1.25:0.65 by weight (Table 1).

iii) **Preparation of rock mass specimens**

The rock mass specimens were formed by arranging elemental blocks of dimensions 25 mm x 25 mm x 75 mm each. The size of the rock mass specimen was 750 mm x750 mm x150 mm, and approximately1800 to 2000 elemental blocks were used for the purpose (Fig. 1). The prepared rock mass specimen had two sets of joints i.e. horizontal and vertical.

Table 1- Engineering and physical properties of model material

S. No.	Property	Value
1	Dry density (kN/m3)	18.5
2	Porosity (%)	36.2
3	Specific gravity	2.5
4	UCS, σ_{ci} (MPa)	7.0
5	Brazilian strength, σ_{ti} (MPa)	1.3
6	Intact rock cohesion, c_i (MPa)	2.0
7	Intact rock friction angle, $\phi_i°$	33.0
8	Tangent modulus of intact rock, E_i (MPa)	2200.0
9	Friction angle along the joints, $\phi_j°$	39.0
10	Deere and Miller classification (1966)	EM

iv) Formation of D-shaped model of underground opening

Several trials were undertaken to simulate an opening being excavated in the field. In initial trials, the rock mass specimen was prepared with specified joint geometry (Rao, 2005). A small drilling machine was then used to excavate the opening in the stressed mass (Fig. 2). This procedure, though much closer to the real field situation, proved to be unsuccessful as it was extremely difficult to control and measure the closure of the drilled opening. Finally, it was decided not to drill the opening in the stressed mass, and rather leave a cavity (along with wall and roof supports) in the mass while preparing the rock mass specimen (Fig. 1). The opening model so generated was 150 mm wide x 215 mm high. The rigid supports contained load cells to measure support pressure. A nut-bolt arrangement was made to allow the rigid supports to displace (Fig. 3).

Fig. 2 - Initial trials of simulating excavation in a rock mass (Rao, 2005)

v) Generation of in situ stresses, simulation of excavation and monitoring of closure and internal support pressure

The tests were conducted for three combinations of in situ horizontal and vertical stresses (Table 2). The ratio of horizontal to vertical in situ stress was varies between 1.00, 1.33 and 2.00 respectively. The details on application of in situ stresses, simulation of excavation and monitoring of closure and internal support pressure are presented in detail in the subsequent sections.

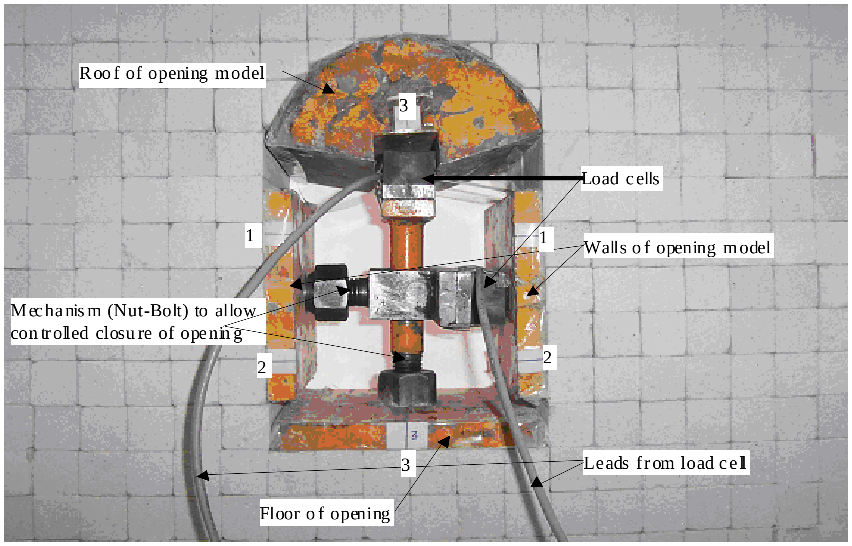

Fig. 3 - Nun-bolt arrangement for allowing displacements of supports

Table 2 - Joint configurations and in-situ stress conditions

S.No.	Joint orientation	Horizontal in situ stress, S_h (MPa)	Vertical in situ stress, S_v (MPa)	Stress ratio, S_h/S_v
1	00/90°	1.72	1.72	1.00
2	00/90°	2.13	1.60	1.33
3	00/90°	2.13	1.07	2.00

2.1 Generation of Stresses

The in situ stresses were generated by applying loads to the rock mass specimen simultaneously in both the directions using hydraulic jacks. Desired horizontal and vertical stresses, S_h and S_v were attained in five to six load increments. When the first load increment was applied, the horizontal and vertical deformations of rock mass were measured using dial gauges HD_1, HD_2, VD_1 and VD_2 (Fig. 1). The loads were maintained constant until the dial gauge readings stabilised. Generally it took approximately about one hour to achieve the steady state. These readings represented the deformations of the rock mass due to the applied stress increments. The support pressures were found to increase gradually with applied load and practically attained the same value. It is possible that some movement of supports might have occurred during the application of load. Subsequent increments of loads were then applied and the corresponding deformations of rock mass specimen were recorded. These steps were followed until the maximum values of horizontal and vertical stresses, S_h and S_v were achieved. The results were obtained in the form of plots of in situ stress versus boundary displacements of the rock mass along the two boundaries (Fig. 4). It is noted that large displacements occurred during first loading step due to seating effect. The straight lines in the plots, therefore, do no pass through origin. Incremental stresses and deformations from these plots were used to obtain the rock mass moduli (E_{0h} and E_{0v}). Poisson's ratio was assumed to be 0.3 in all the cases. The values were very low for first stress increment due to seating effect. These values were excluded and the average values for subsequent stress increments have been considered to determine the representative values of the rock mass moduli, E_{0h} and E_{0v} for different tests. Summary of the analysis of these tests data is presented in Table 3.

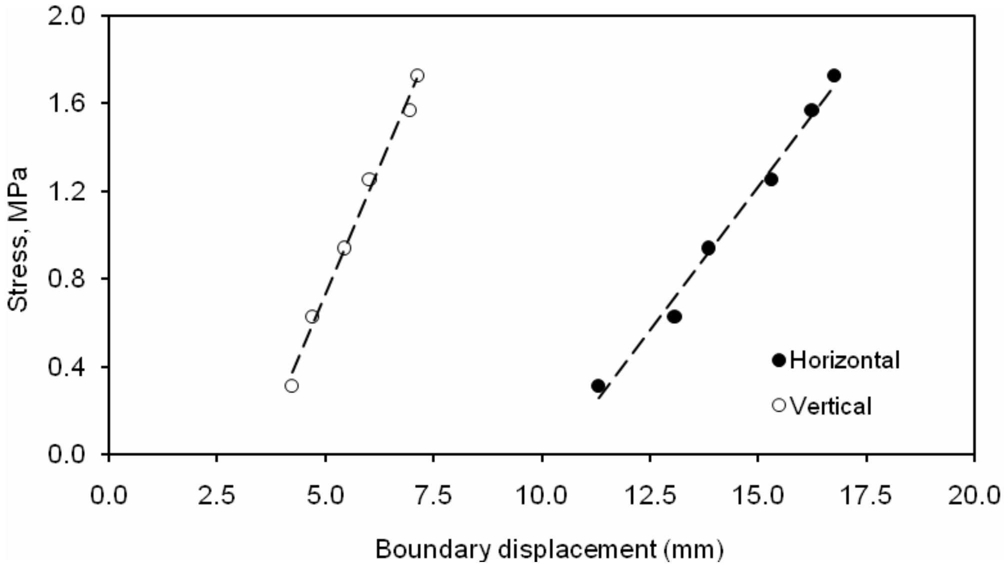

Fig. 4 – In situ vs boundary displacement plots for Test 1 (00/90°; SR-1.0)

Table 3 - Summary of the representative values of rock mass moduli E_{0h} and E_{0v} for different tests

Test No.	Joint Orientation	Stress Ratio	S_h	S_v	E_{0h} (MPa)	E_{0v} (MPa)	E_{mean} (MPa)
1	00/90	1	1.72	1.72	164.8	231.7	198.25
2	00/90	1.33	2.13	1.6	154.5	182.1	168.3
3	00/90	2	2.13	1.07	144.2	125.4	134.8

2.2 Closure and Corresponding Internal Support Pressure

The final stresses S_h and S_v were maintained for 24 hours, and the internal support pressure (p_{ih} and p_{iv}) were monitored during this period and were found to stabilise within 24 hours. These values were assumed to represent the initial internal pressures corresponding to zero closure of the opening.

In the second phase of loading, the opening periphery was subjected to small incremental closures in both the horizontal and vertical directions by displacing the supports using the nut and bolt arrangement. The internal support pressures were found to fall instantly as a result of these closures, however, the pressures were found to increase with time and then stabilise approximately within 24 hours. The stabilised support pressures were lower than the initial ones as the supports had displaced from the initial positions. The support pressures were recorded along with the respective closures in the wall and roof of the opening.

Subsequent increments of closures were then allowed in both the directions simultaneously, and the stabilised internal support pressures were again recorded after

24 hours. These steps were repeated until the support pressures had reduced to zero i.e. equivalent to the condition of an unsupported opening. A typical plot of support pressure variation with time is presented in Fig. 5. Generally 15 to 20 days were required to complete one test. The results of these tests have been summarised in the form of ground response curves and are presented in Figs. 6 a-c.

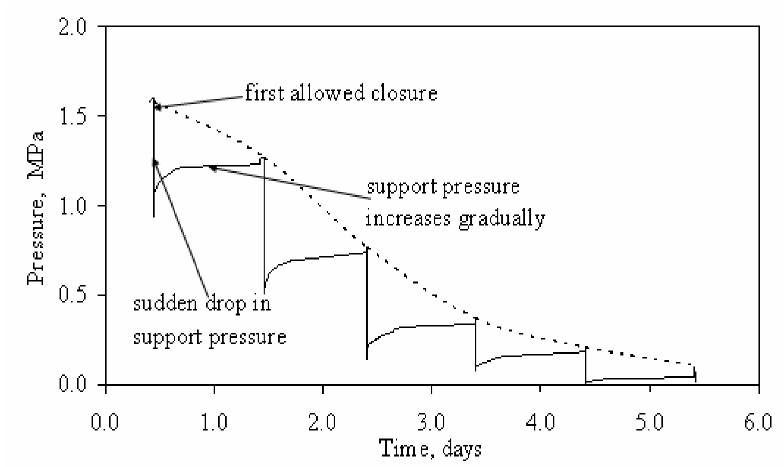

Fig. 5 - Variation of pressure on wall of the opening with time

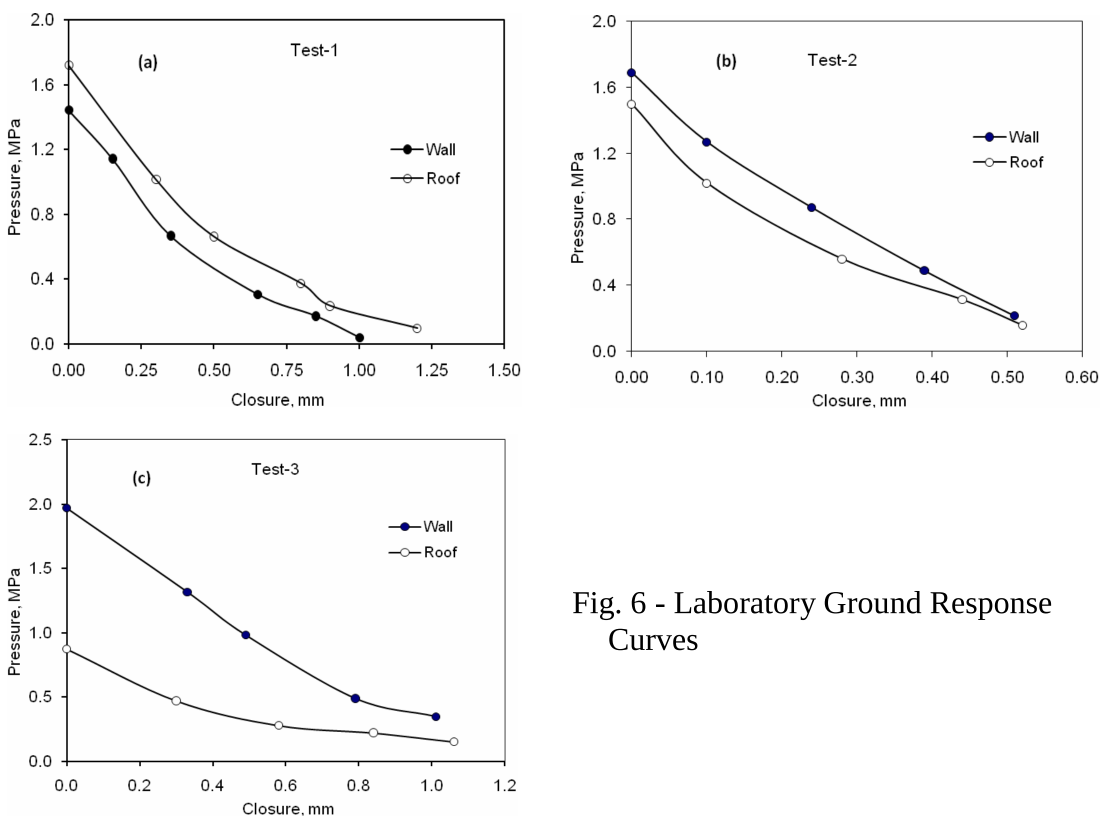

Fig. 6 - Laboratory Ground Response Curves

3. PREDICTION OF CLOSURE

3.1 Closed form Solutions

Closed form solution for estimating closure of a circular opening subjected to horizontal internal pressure on a part of periphery of the opening and subtending at an angle, α at the centre of the opening (Fig. 7), is available in Bray (1987). Assuming the principle of superposition to be valid for elastic ground condition, similar expressions will hold good for vertical internal pressure also. These solutions along with standard expressions for the closure of an opening subject to far field stresses (Singh and Goel, 2006) were used to predict the side wall and roof displacements of the opening tested (Fig. 1.). Only those closures which pertain to elastic ground condition have been considered. Elasto-plastic ground conditions are beyond the scope of the present paper.

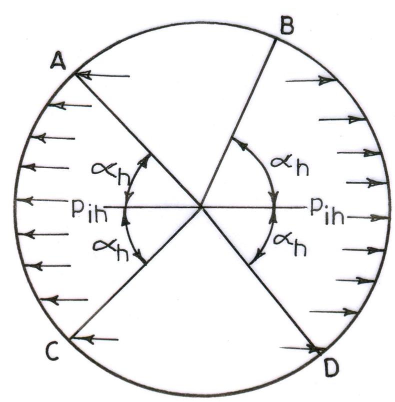

Fig 7 - Circular opening subjected to horizontal internal pressure (Bray,1987)

Consider a circular underground opening subjected to far-field stresses, S_h and S_v, and uniform internal pressures, p_{ih} and p_{iv} in horizontal and vertical directions respectively. Also consider that the rock mass behaves elastically and the principle of superposition for stresses and strains is valid. The resulting circumferential stress at the wall ($\theta = 0°$) and the roof ($\theta = 90°$) may be obtained as:

$$\sigma_{\theta wall} = p_{ih} - \frac{4}{\pi} p_{iv}\, \alpha_v + 3S_v - S_h \tag{1}$$

$$\sigma_{\theta roof} = p_{iv} - \frac{4}{\pi} p_{ih}\, \alpha_h + 3S_h - S_v \tag{2}$$

where $\sigma_{\theta wall}$ is the circumferential stress at the wall; $\sigma_{\theta roof}$ is the circumferential stress at the roof; α_h is half of the angle subtended at the centre of the opening by the arc of the circle on which the internal horizontal pressure, p_{ih} is acting (Fig. 7) and α_v is half of the angle subtended at the centre by an arc of the circle on which the internal vertical pressure, p_{iv} is acting.

The total horizontal displacement of the wall for given internal pressures and far-field in situ stresses may be obtained by considering the following four components of displacement.

i. Displacement due to horizontal internal pressure, p_{ih}

$$u_{h1} = \frac{2p_{ih}a}{\pi E_{0h}}\left(1-\nu^2\right)\left[\sin\alpha_h \left|\frac{1+\cos\alpha_h}{1-\cos\alpha_h}\right| + 2\alpha_h\right] \tag{3}$$

ii. Displacement due to vertical internal pressure, p_{iv}

$$u_{h2} = -\frac{p_{iv}a\,(1+\nu)}{E_{0v}}\left(1-2\nu\right)\sin\alpha_v \tag{4}$$

iii. Displacement due to far field in situ stress, S_h

$$u_{h3} = -\frac{2S_h a}{E_{0h}}\left(1-\nu^2\right) \tag{5}$$

iv. Displacement due to far field in situ stress, S_v

$$u_{h4} = \frac{S_v a\,(1+\nu)}{E_{0v}}\left(1-2\nu\right) \tag{6}$$

where u_{h1}, u_{h2}, u_{h3} and u_{h4} are the horizontal components of displacement due to p_{ih}, p_{iv}, S_h and S_v respectively. The displacements are taken positive with increasing radial distance. The total horizontal displacement for given p_{ih}, p_{iv}, S_h and S_v is then computed as

$$u_h = u_{h1} + u_{h2} + u_{h3} + u_{h4} \tag{7}$$

Similarly, the total vertical displacement of the roof subjected to given p_{ih}, p_{iv}, S_h and S_v is obtained as:

$$u_v = u_{v1}+u_{v2}+u_{v3}+u_{v4} \tag{8}$$

where

$$u_{v1} = -\frac{p_{ih}a\,(1+\nu)}{E_{0h}}\left(1-2\nu\right)\sin\alpha_h \tag{9}$$

$$u_{v2} = \frac{2p_{iv}a}{\pi E_{0v}}\left(1-\nu^2\right)\left[\sin\alpha_v \left|\frac{1+\cos\alpha_v}{1-\cos\alpha_v}\right| + 2\alpha_v\right] \tag{10}$$

$$u_{v3} = \frac{S_h a\,(1+\nu)}{E_{0h}}\left(1-2\nu\right) \tag{11}$$

$$u_{v4} = -\frac{2S_v a}{E_{0v}}\left(1-\nu^2\right) \tag{12}$$

where u_v is the displacement of the roof.

3.2 Step by Step Procedure to Compute Closure

An equivalent circular opening was considered for application of the closed form solutions to the model tests. The size of the equivalent opening was obtained by equating its surface area with the surface area of the model opening. Plane stress conditions were assumed in the analysis. The following step-by-step procedure was followed to predict the closure of the side wall of the model tunnel:

- Say, the closure is required for the walls of the equivalent circular opening for the various values of internal pressures observed during the experiment.
- Consider the initial values of horizontal and vertical internal pressures in the equivalent circular opening.
- Compute circumferential stress at the wall by using Eq. 1
- Check if the circumferential stress is less than the strength of the rock mass for the present elastic solution to be valid. If the circumferential stress exceeds the confined strength of the rock mass, the elastic solution will be invalid and computations will be terminated. This step is explained in detail in the next section.
- Compute total radial closure of the wall due to far field stresses and internal pressures using Eq. 7. While applying far field stresses in the laboratory it is likely that some displacements of the supports might have occurred. The closure corresponding to the initial internal pressure was considered zero. The closure obtained above in this step, therefore, provides the correction to be applied in each calculated closure value in subsequent steps.
- Consider the next pair of internal pressures and calculate the total radial closure. Apply the correction as mentioned in the previous step to get the predicted closure corresponding to the pair of internal pressures.
- The procedure is continued for different values of subsequent pairs of internal pressures until the internal pressure values are zero (self supporting opening), or until the computed strength of the rock mass at the periphery is exceeded by the circumferential stress (development of a plastic zone). The elastic analysis is now terminated.

The same step-by-step procedure was adopted to predict the closure of roof also for various values of internal pressure.

3.3 Determination of Rock Mass Strength in Confined State

The strength of confined rock mass at the applicable internal pressure is required to check if the rock mass at the opening periphery is in elastic state or not. A simple parabolic strength criterion as proposed by Singh and Singh (2004) and Singh and Rao (2005a) was used for this purpose. The strength criterion is expressed as:

$$\sigma_1 = \sigma_{cj} + (1-2A\sigma_{ci})\sigma_3 + A\,(\sigma_3)^2;\;\; \sigma_3 \le \sigma_{ci} \tag{13}$$

where σ_3 and σ_1 are the minor and major principal stresses at failure; σ_{ci} and σ_{cj} are the UCS values of the intact rock and the jointed rock respectively; and A is an empirical parameter (Singh and Rao, 2005a) expressed as:

$$A = -1.23\,(\sigma_{ci})^{-0.77} \tag{14}$$

It may be noted that for walls, the tangential stress will be in the vertical direction and hence the UCS of rock mass, σ_{cj} should also be obtained in vertical direction. The UCS of the rock mass in vertical direction, σ_{cjv} was obtained from the correlation suggested by Singh and Rao (2005b) as:

$$\sigma_{cjv} = \sigma_{ci}\left(\frac{E_{0v}}{E_i}\right)^{0.63} \tag{15}$$

where E_{0v} is the rock mass modulus in vertical direction under uniaxial loading condition. The UCS of the rock mass in horizontal direction σ_{cjh} may also be obtained in similar manner. By using expressions 14 and 15 and the Eq. 13, the rock mass strength values were obtained for the side wall and the roof. These values were used to decide if the rock mass was in elastic condition or not.

4. APPLICATION OF CLOSED FORM SOLUTIONS TO THE PRESENT STUDY

4.1 Closure Prediction by using Pressure Independent Rock Mass Modulus

The values of rock mass moduli, E_{0h} and E_{0v} were obtained from initial compression tests when in situ stresses were applied (Table 3). The step-by-step procedure discussed in the previous section was employed to determine the closure of the opening for various internal support pressures used in this study. To compare the predictions with the laboratory test results, all the values of predicted closures were plotted against experimental closures and are presented in Fig. 8. It can easily be inferred from these plots that there is a large difference between the observed and the predicted values of closures for the wall and roof of the model of the opening. The observed ground response curves are non-linear and not linear as expected from the elastic theory.

4.2 Closure Prediction by using Pressure Dependent Rock Mass Modulus

A comparison of predicted closures with those observed in the laboratory indicates that the closures have been over predicted. This follows that rock modulus mobilised in generating closure is much higher than that used in the computations. In the present analysis, a constant value of rock mass modulus derived out of compression tests was used. There could be two reasons of mobilising higher rock mass modulus in the mass. The first one is the difference between the loading and the unloading moduli of rock mass. The closure of the opening is initiated due to re-distribution of stresses around the periphery of the opening. There is loosening and unloading of the rock mass which results in the closure of the opening and the rock mass modulus under unloading condition is much higher than that obtained during compression tests. This difference occurs due to in-elastic behaviour of the joints. The second reason is that the modulus itself may behave in a non-linear manner with confining pressure (Kulhawy, 1975; Duncan and Chang, 1970).

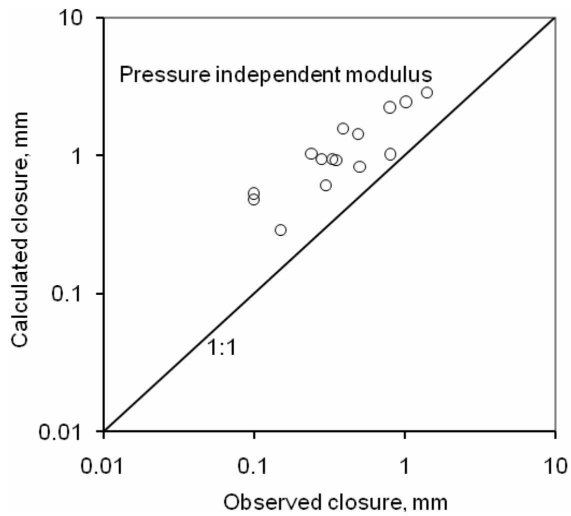

Fig. 8 - Comparison of predicted and observed closures considering pressure independent modulus

To improve predictions, it was decided to use an empirical approach wherein the rock mass modulus is taken as a function of confining pressure acting over the mass. An expression similar to that suggested by Duncan and Chang (1970) was used to determine the pressure dependent modulus of the rock mass. The expression is suggested as:

$$E_{h,v} = \left(\frac{p_{ih,iv} + p_a}{p_a} \right)^{\beta_{h,v}} E_{0h,0v} \tag{16}$$

where E_h is the pressure dependent modulus in the horizontal direction; p_{ih} is the internal pressure in the horizontal direction; E_{0h} is the rock mass modulus under uniaxial compression (i.e. at confining pressure equal to the atmospheric pressure) in the horizontal direction; β_h is an empirical modulus exponent to be determined from back analysis. The atmospheric pressure p_a was taken equal to 0.105 MPa. Similar expression has been assumed to compute the pressure dependent rock mass modulus in the vertical direction, E_v. This empirical approach takes care of the stress anisotropy.

The procedure for predicting the closure was modified by taking into account the pressure dependent modulus. Several trial values of the indices β_h and β_v were employed to optimize the predictions. The most promising predictions were found to have been achieved by using the values presented in Table 4. The index, β_h has been found to vary between 0.36 to 0.68 with an average of 0.56. The index, β_v has been found to vary between 0.25 and 0.50 with an average of 0.34. On an average, the index, β_v can be taken roughly equal to 0.62 times the index β_h.

Table 4 - Optimal values of indices β_h and β_v

Test No.	Stress ratio	Index β_h	Index β_v	β_v/β_h
1	1	0.36	0.25	0.69
2	1.33	0.68	0.50	0.74
3	2	0.63	0.27	0.43
Average		**0.56**	**0.34**	**0.62**

A comparison of all the predicted closures using pressure dependent modulus with observed laboratory closures is presented in Fig. 9. It can be observed that there is a substantial improvement in the prediction and most of the points are closed to a line drawn with 1:1 slope. It is, therefore, concluded that substantial improvement in the prediction of closure of openings can be achieved by using the empirical concept of pressure dependent modulus of the rock mass.

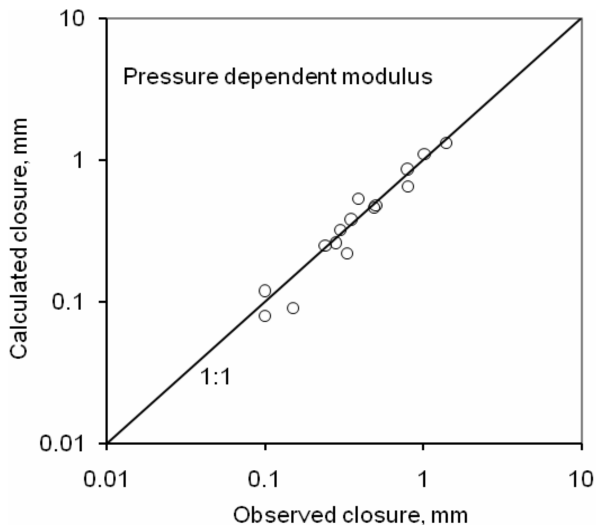

Fig 9. Comparison of predicted and observed closures considering pressure dependent modulus

In the field, rock mass modulus may be estimated either through uniaxial jacking tests or by use of the classification approaches in which it is estimated as a function of RMR (Bieniawski, 1978; Serafim and Pereira, 1983) or rock mass quality index Q (Barton, 2002) or Joint Factor (Ramamurthy, 1993; Singh et al., 2002). Field tests are very expensive, time consuming and some times not feasible also. In such situations the designers have no choice but to use classification approach. These techniques generally represent the rock masses which are unconfined in nature. It is suggested that the Eq. 16 along with the average value of the indices, β_h and β_v equal to 0.56 and 0.34 respectively, may be used to get the pressure dependent modulus of the rock mass for analysing closure of openings.

5. SUMMARY AND CONCLUSIONS

Rock mass modulus is one of the most important parameters which influence the closure of underground openings. Jointed rocks in the field are generally subjected to in situ stresses and excavation of opening results in re-distribution of these stresses around the periphery of the opening. The current practice of assessing the closure of the openings involves the use of a pressure independent modulus, which is obtained from uniaxial jacking tests or the classification approaches. Using results of a laboratory testing programme, it has been shown that, an assumption of a pressure independent rock mass modulus obtained from unconfined conditions, results in the over prediction of the closure of the opening. It is, therefore, suggested that the rock mass modulus should be corrected for the confining pressure acting over the rock mass. An empirical expression, on lines similar to Duncan and Chang (1970), to determine the pressure dependent rock mass modulus has been suggested in this study. It is concluded that the predictions of the closure of underground openings can be improved substantially by using the pressure dependent modulus of the rock mass.

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