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Nonlinear Triaxial Strength Criterion of Inherently Anisotropic Rocks

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ABSTRACT

Many of the rocks encountered at tunnel and underground power house sites in Himalayan region are observed to be foliated. The triaxial strength behaviour of such rocks varies substantially with the loading direction. The triaxial strength also varies in a non-linear manner with increase in confining pressure. While analyzing strength behaviour of underground openings in such rocks, the geologists and engineers need a strength criterion to capture non-linearity as well as the anisotropy in the triaxial strength behaviour of the rocks. The present paper discusses a strength criterion termed as 'Modified Mohr-Coulomb Criterion (MMC)' which can be used to describe the behaviour of intact anisotropic rocks under triaxial stress condition. A simple criterion has been deduced by modifying the conventional linear Mohr-Coulomb (MC) strength criterion by invoking critical state concepts for rocks. The applicability of the suggested criterion has been evaluated by applying it to a data base comprising of more than 1140 triaxial tests conducted world-wide on anisotropic rocks. Further the predictive capabilities of the proposed criterion have been evaluated by determining the error in estimation of triaxial strength if only few triaxial test data are available for determining the criterion parameters. It is concluded that reasonable estimates of the triaxial strength of anisotropic rock can be made through the proposed criterion by using minimum amount of triaxial test data available.

Keywords: Mohr-Coulomb theory; Critical state mechanics; Triaxial tests; Anisotropic rocks; Modified Mohr-Coulomb criterion; Critical confining pressure

1. INTRODUCTION

Hydropower development activities invariably deal with design and construction of tunnels and underground powerhouses in rocks. In Himalayan region it is very common to come across the rocks such as shale, slate, gneiss, schist and phyllites which are laminated and foliated. The strength behaviour of these rocks varies substantially with change in loading direction. While analysing and designing underground openings in such rocks the geologists and engineers need an adequate understanding of the strength behaviour of inherently anisotropic rocks subject to given stress conditions. It is known that the strength of rocks varies non-linearly with increase in confining pressure. In the present paper, the conventional linear Mohr-Coulomb (MC) criterion is modified and a 'Modified Mohr-Coulomb (MMC)' criterion is suggested to incorporate a non-linear strength behaviour of inherently anisotropic rocks.

2. MODIFIED MOHR-COLULOMB (MMC) CRITERION FOR ANISOTROPIC ROCKS

The strength criterion is deduced from the linear Mohr-Coulomb failure criterion (Singh et al., 2015). The conventional linear Mohr-Coulomb criterion for triaxial strength of a rock is written as:

$$\sigma_1 = \sigma_{c\beta} + \frac{1 + \text{Sin}\ \phi_{\beta 0}}{1 - \text{Sin}\ \phi_{\beta 0}} \sigma_3 \tag{1}$$

where σ_1, σ_3 are major and minor principal stresses at failure, and

$$\sigma_{c\beta} = \frac{2\, c_{\beta o} \cos\phi_{\beta 0}}{1 - \sin\ \phi_{\beta 0}} \tag{2}$$

= UCS of the anisotropic rock, with planes of anisotropy oriented at an angle of β from major principal stress direction. The $c_{\beta 0}$ and $\phi_{\beta 0}$ are the Mohr-Coulomb shear strength parameters obtained by conducting triaxial strength tests on rock specimens at very low confining pressure ($\sigma_3 \rightarrow 0$).

Figure 1 shows the linear Mohr-Coulomb (MC) and the proposed non-linear (MMC) strength criterion in the form of differential stress at failure vs. confining pressure plot. The linear form of the MC criterion may be written from Eq. 1 as:

$$\sigma_1 - \sigma_3 = \sigma_{c\beta} + \frac{2 \sin\ \phi_{\beta 0}}{1 - \sin\ \phi_{\beta 0}} \sigma_3 \tag{3}$$

The proposed MMC criterion is nonlinear and concave downward. To get the equation of the non-liner criterion, a second degree term of σ_3 is introduced in the strength criterion. The non-linear MMC criterion is written as:

$$\sigma_1 - \sigma_3 = \sigma_{c\beta} + \frac{2 \sin\ \phi_{\beta 0}}{1 - \sin\ \phi_{\beta 0}} \sigma_3 - A'\sigma_3^2 \text{ for } 0 \le \sigma_3 \le \sigma_{crt} \tag{4}$$

where σ_{crt} = critical confining pressure for the rock, A' is an empirical constant which defines the shape of the strength criterion.

To get the empirical constant A', the critical state concept of rocks (Barton, 1976) is invoked. The concept states that *"critical state for an initially intact rock is defined as the stress condition under which Mohr envelope of peak shear strength of the rocks reaches a point of zero gradient. This condition represents the maximum possible shear strength of the rock. For each rock, there will be a critical effective confining pressure above which the shear strength cannot be made to increase"*.

The concept was initially suggested for intact isotropic rocks. It is however seen that the anisotropic rocks also follow the critical state concept.

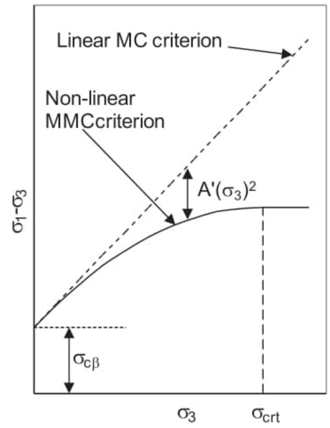


Fig. 1 - Linear MC and non-linear MMC criteria (Singh et al., 2015)

At critical state ($\sigma_3 \Rightarrow \sigma_{crt}$), the deviator stress at failure will become constant and the gradient of Mohr failure envelope will be zero.

Differentiating the nonlinear Eq. 4,

$$\frac{\partial}{\partial \sigma_3}(\sigma_1 - \sigma_3) = \frac{2\sin\phi_{\beta 0}}{1-\sin\phi_{\beta 0}} - 2A'\sigma_3 \tag{5}$$

Putting boundary condition $\frac{\partial}{\partial \sigma_3}(\sigma_1 - \sigma_3) = 0$ for $\sigma_3 \Rightarrow \sigma_{crt}$

$$A' = \frac{1}{\sigma_{crt}} \frac{\sin\phi_{\beta 0}}{1-\sin\phi_{\beta 0}} \tag{6}$$

The nonlinear form of the criterion becomes:

$$(\sigma_1 - \sigma_3) = \sigma_{c\beta} + \frac{2\sin\phi_{\beta 0}}{1-\sin\phi_{\beta 0}}\sigma_3 - \frac{1}{\sigma_{crt}}\frac{\sin\phi_{\beta 0}}{(1-\sin\phi_{\beta 0})}\sigma_3^2 \text{ for } 0 \le \sigma_3 \le \sigma_{crt} \tag{7}$$

For confining pressure range $\sigma_3 > \sigma_{crt}$, the criterion takes the following form:

$$(\sigma_1 - \sigma_3) = \sigma_{c\beta} + \frac{\sin\phi_{\beta 0}}{1-\sin\phi_{\beta 0}}\sigma_{crt} \quad \text{for } \sigma_3 > \sigma_{crt} \tag{8}$$

To apply the proposed criterion (Eqs. 7& 8), the UCS $\sigma_{c\beta}$, of the rock at a given orientation β, should be available. This UCS may be obtained by conducting UCS tests at orientations β = 0, 30 and 90° and $\sigma_{c\beta}$ may be obtained by using correlations available in literature (Nasseri et al., 2003). For using the MMC criterion two parameters i.e. $\phi_{\beta 0}$ and σ_{crt} are required. The form of the MMC criterion given above is termed 'two parameter form' as, two parameters $\phi_{\beta 0}$ and σ_{crt} are required to apply this expression. In previous studies for intact isotropic and jointed rocks (Singh and Singh, 2005; Singh et al., 2011; Singh and Singh, 2012), the critical confining pressure was found to be approximately equal to the UCS of the rock.

3. PERFORMANCE EVALUATION OF THE PROPOSED CRITERION

From practical point of view a criterion should satisfy the following requirements: a) The criterion should be simple in use. b) The criterion parameters should carry physical meaning; the designer should be able to obtain them with minimum number of tests at low confining pressure or should be able to roughly correlate the parameters with his past experience. c) The criterion should show excellent goodness of fit to the existing triaxial test data base.

To evaluate the performance of the proposed criterion, a triaxial test data base was compiled (Singh et al., 2015). The following indices were employed to evaluate the performance of the proposed criterion.

(i) Percent Error

For each individual triaxial test data point the percent error in prediction was calculated as:

$$pe = \left(\frac{\sigma_{1cal} - \sigma_{1exp}}{\sigma_{1exp}}\right) \text{x } 100 \text{ percent} \tag{9}$$

where, σ_{1exp} is the experimental and σ_{1cal} is the predicted value of the major principal stress at failure for given confining pressure, σ_3.

(ii) Coefficient of Accordance (COA)

For each set of triaxial tests conducted for a given discontinuity orientation β, the index COA was obtained as:

$$\psi^2 = \frac{\sum(\sigma_{1exp} - \sigma_{1cal})^2}{\sum(\sigma_{1exp} - \sigma_{1av})^2} \tag{10}$$

where, σ_{1av} is the average of the experimental σ_1 values for the triaxial data set. A lower value of ψ^2 indicates better prediction.

(iii) Average Absolute Relative Error Percentage

The overall error in prediction for the entire data base was obtained as:

$$AAREP = \frac{\sum_{i=1}^{N}\left|\frac{\sigma_{1cal} - \sigma_{1exp}}{\sigma_{1exp}}\right|}{N} \text{x } 100\% \tag{11}$$

where, N is the total number of triaxial data points in the data base.

In addition to the error measurements defined above, the regression R-square value (R^2) was also used to assess the correlation between the experimental (σ_{1exp}) and the predicted (σ_{1cal}) values of the database.

The data base (Singh et al., 2015) used in this study comprises triaxial test results on 38 rock types with total number of 255 UCS and 1141 triaxial tests. The data base also includes results from an experimental study conducted in house at IIT Roorkee (Kumar 2006). The brief summary of the experimental study is given below.

3.1 Laboratory Tests at IIT Roorkee

A series of uniaxial and triaxial tests was performed by Kumar (2006) on intact anisotropic rocks at IIT Roorkee. The tests were conducted on three rock types namely phyllite, state and orthoquartzite obtained from three different project sites (Fig. 2, Tables 1 & 2).

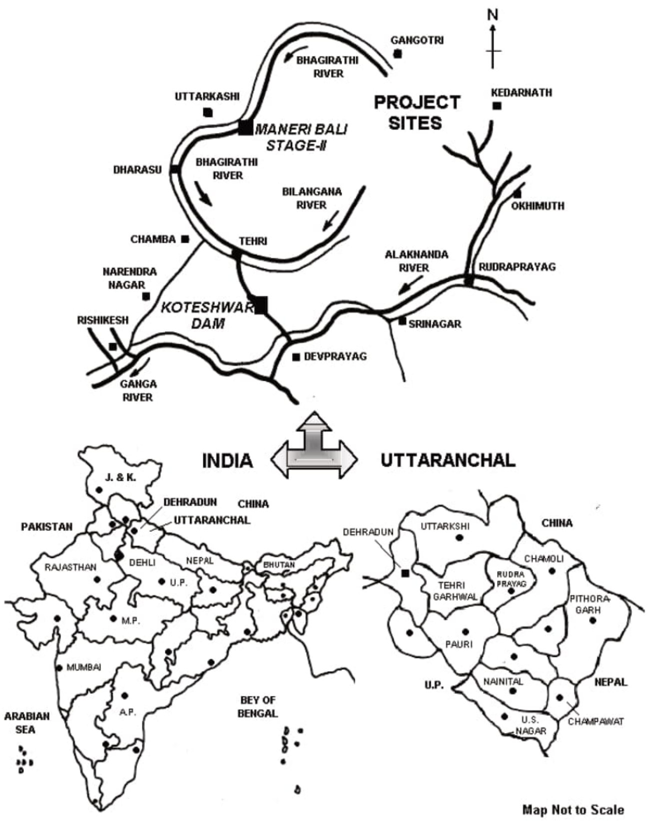


Fig. 2 - Project sites from where rocks were extracted (Kumar, 2006)

Table 1 - Details of rocks tested (Kumar, 2006)

Rock type	Location	Project details
Phyllite	Koteshwar, Uttarakhand, India	Koteshwar hydroelectric project: 100 m high concrete dam on river Bhagirathi
Slate & orthoquartzite	Maneri Bhali Hydro Electric power project Stage-II, at Joshiyara near Uttarkashi, Uttarakhand	81m long barrage across river Bhagirathi at Joshiyara; 6m equivalent diameter and 16 km long horse shoe shaped tunnel; 304 MW surface power house at Dharasu.

Table 2 - Physical properties of rocks (Kumar, 2006)

Property	Rock Type		
	Phyllite	Slate	Orthoquartzite
Dry unit weight (γ_d), kN/m^3	27.00	26.16	27.16
Saturated unit weight (γ_{sat}), kN/m^3	27.08	26.31	27.24
Specific gravity (G)	2.79	2.69	2.80
Porosity (n), %	0.81	1.56	0.86
Saturated water content, %	0.29	0.58	0.31

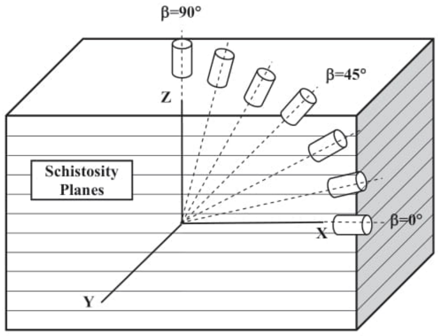
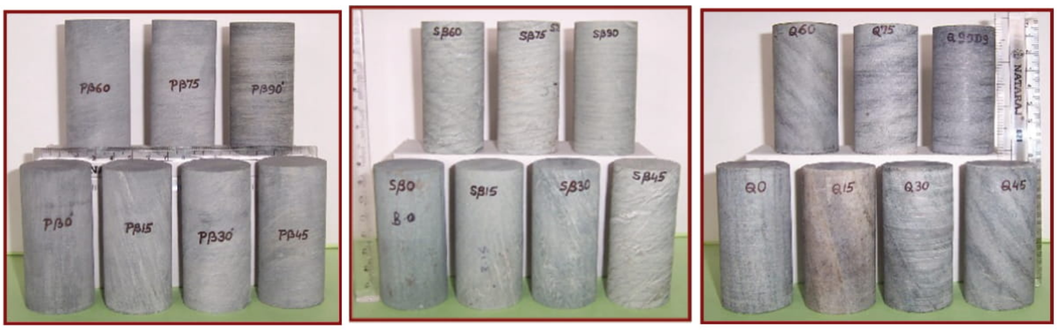


Fig. 3 - Sampling directions for drilling cores (Kumar, 2006)



Phyllite Slate Orhoquartzite

Fig. 4 - Typical specimens for testing (Kumar, 2006)

The specimens were extracted from the chunks of the rocks through drilling at different directions with respect to schistocity planes. The rock specimens having 38 mm diameter with height-to-diameter ration 2 were used. The specimens were drilled at different schistosity angles i.e. β=0°, 15°, 30°, 45°, 60°, 75° and 90° respectively (Fig. 3). Typical specimens are shown in Fig. 4. The tests were conducted under normal and saturated conditions. For saturation, the specimens were submerged in water for a couple of months. Cycles of heating and cooling were applied and vacuum was applied for removing trapped air. The weight of surface dry specimens was monitored till it stabilized.

The rocks, for each orientation β, were tested under uniaxial and triaxial stress conditions. For each orientation three uniaxial compression tests were performed and average value was considered for the UCS. During tests, axial load, axial strain and lateral strain were recorded. Electrical strain gauges were used to measure the strains. A 3.0 MN conventional universal testing machine with an accuracy of ±0.1 kN was used to perform the tests. To minimise end friction, 0.5mm thick Teflon sheets were used at the ends of the specimen. The loading rate was so adjusted that failure occurred within about 5-10 minutes. For triaxial tests, a conventional triaxial compression testing machine with a capacity of applying confining pressure upto 70 MPa was used. The cell body had provision of connection of strain gauge wires from inside and the wire leads of data acquisition system from outside. The triaxial tests were performed at confining pressures (σ_3) of 5, 15, 30 and 60 MPa respectively for each orientation. All the tests were performed under saturated and dry conditions.

3.2 Goodness of Fit of the MMC

One of the important features of any strength criterion is that it should fit into the data base. It is assumed that for any given orientation β, the UCS is available as experimental data; and $\phi_{\beta 0}$ and σ_{crt} are the criterion parameters that are obtained by fitting available triaxial test data. To evaluate goodness of fit of the proposed MMC, the criterion parameters $\phi_{\beta 0}$ and σ_{crt} for a given set of data were obtained by least square method i.e. by minimising the sum of squares of the deviations for assumed set of parameters. A computer program was written for this purpose. The optimal values of parameters, σ_{crt} and $\phi_{\beta 0}$ were now used to predict the triaxial strength values for given confining pressure values. The comparison of the calculated and experimental values for the three rocks is shown in Fig. 5a. The average COA and AAREP were observed to be 0.010 and 3.91% respectively. An excellent agreement is therefore observed between experimental and the calculated values.

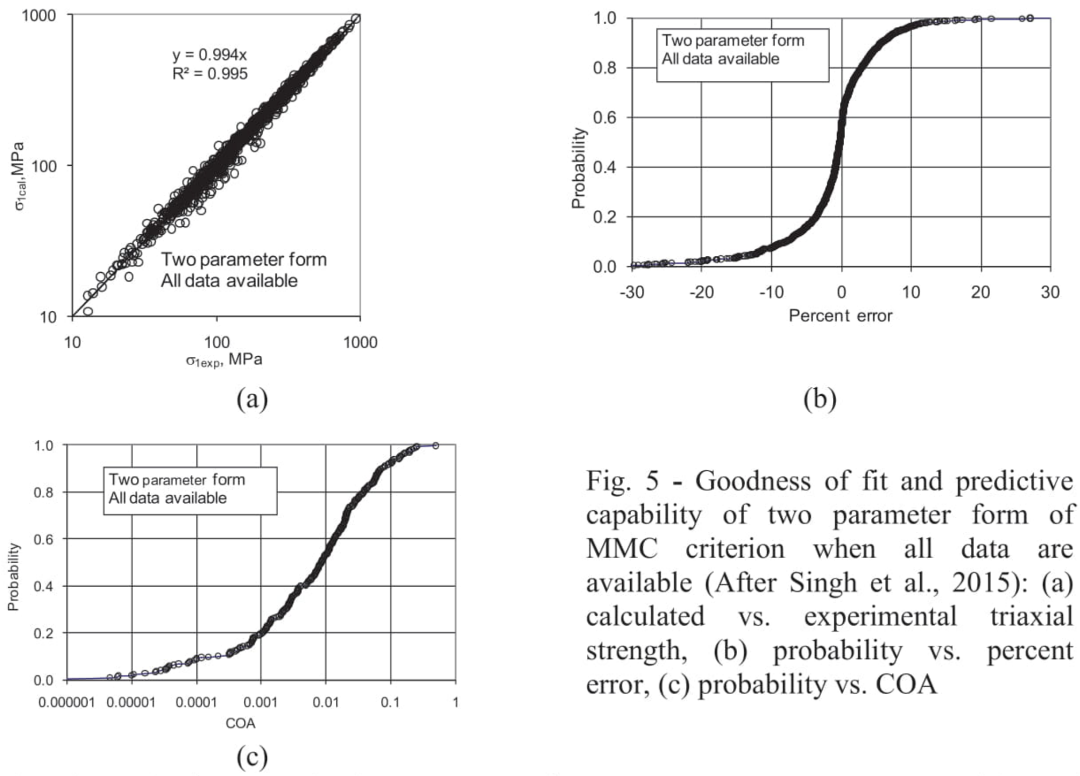


Fig. 5 - Goodness of fit and predictive capability of two parameter form of MMC criterion when all data are available (After Singh et al., 2015): (a) calculated vs. experimental triaxial strength, (b) probability vs. percent error, (c) probability vs. COA

Using the optimal set of criterion parameters the percent error 'pe' was computed for each data point of the data base. The cumulative distribution function of these errors is shown in Fig. 5b. The plot indicates that probability of error in prediction to lie within ± 20% is

0.9737. The very high probability of reasonable error to be within 20% indicates an excellent fitness of the proposed model to the data base.

The COA was also obtained for all the 255 data sets. The probability distribution function of the COA is shown in Fig. 5c. It is seen that the probability of COA to be less than or equal to 0.1 is 0.9219 which is very high. The index AAREP, that reflects overall error in prediction, is found to be 4.20% and is extremely low.

It can be inferred from the above analysis that the proposed criterion fits excellently in the compiled data base.

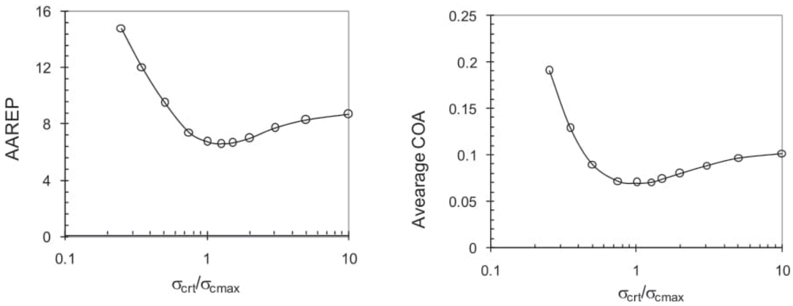


Fig. 6 - Variation of AAREP and COA with critical confining pressure (After Singh et al., 2015)

4. SINGLE PARAMETER FORM OF THE MMC CRITERION

Ideally the criterion parameter σ_{crt} should be obtained from laboratory triaxial tests. In reality it may however not always be feasible and a rough assessment of this parameter will be required. Adequate information is not available in the literature on this aspect. In absence of such information, it was decided to back analyse the results of the data base to statistically arrive at most probable value of the critical confining pressure. For this purpose the statistical error measurements AAREP and COA were employed. Different values of critical confining pressure σ_{crt} were assumed and strength values were predicted. The indices AAREP and COA were obtained. The values of AAREP and COA so obtained were plotted against the normalised σ_{crt} (Fig. 6). The assumed values of σ_{crt} were normalised by dividing them by the UCS of the rock. As UCS depends on orientation β, the maximum UCS, σ_{cmax} was used for normalisation. The plots indicate that minimum error is obtained at critical confining pressure nearly equal to $1.25\sigma_{cmax}$. This statistical analysis indicates that if the critical confining pressure is taken equal to $1.25\sigma_{cmax}$, the proposed criterion is likely to give minimum error. It may be noted that the critical confining pressure may be much higher and will also depend on lithology and many other factors. However the use of critical confining pressure equal to 1.25 σ_{cmax} in the proposed criterion is not likely to introduce errors of engineering significance. It is, therefore, suggested that in absence of sufficient triaxial tests data, an average value of the critical confining pressure may be taken nearly equal to $1.25\sigma_{cmax}$. The 'single parameter form' of the MMC criterion may now be written as:

$$(\sigma_1 - \sigma_3) = \sigma_{c_\beta} + \frac{2\sin\phi_{\beta o}}{1-\sin\phi_{\beta o}}\sigma_3 - \frac{1}{1.25\,\sigma_{cmax}}\frac{\sin\phi_{\beta o}}{(1-\sin\phi_{\beta o})}\sigma_3^2 \text{ for } 0 \le \sigma_3 \le 1.25\,\sigma_{cmax} \tag{12}$$

$$(\sigma_1 - \sigma_3)_{max} = \sigma_{c_\beta} + 1.25\frac{\sin\phi_{\beta o}}{1-\sin\phi_{\beta o}}\sigma_{cmax} \quad \text{for } \sigma_3 > 1.25\,\sigma_{cmax} \tag{13}$$

The above criterion has only single parameter $\phi_{\beta 0}$, which may be obtained by adopting the following steps.

$$A = \frac{\sum(\sigma_1 - \sigma_3 - \sigma_{c\beta})}{\sum(\sigma_3^2 - 2\,\sigma_{crt}\,\sigma_3)} \qquad \text{for } 0 \le \sigma_3 \le 1.25\,\sigma_{cmax} \tag{14}$$

where $\sigma_{crt} = 1.25\,\sigma_{cmax}$ (15)

$$B = -\,2.0\,A\,\sigma_{crt} \tag{16}$$

$$\sin\phi_{\beta 0} = \frac{B}{2+B} \tag{17}$$

It is worth to evaluate the predictive capability of the single parameter form of the MMC criterion, especially when sufficient triaxial test data is not available to fit $\phi_{\beta 0}$. The data base is utilised for evaluation of the predictive capabilities. Three conditions are considered regarding availability of test data (Singh et al., 2015) for fitting the criterion parameter $\phi_{\beta 0}$ namely:

i) if the entire data base is available for fitting $\phi_{\beta 0}$;
ii) when only two triaxial test data are available for fitting $\phi_{\beta 0}$; and
iii) when no triaxial test data is available for fitting $\phi_{\beta 0}$.

For the cases (i) and (ii) the predicted values of σ_1 were calculated from the criterion and the error indices were obtained. The plot between predicted σ_1 values vs. experimental values was obtained (Fig. 7a). Also plots of cumulative distribution of percent error and COA were obtained (Figs. 7b & c). The indices AAREP and COA were also obtained for both the cases.

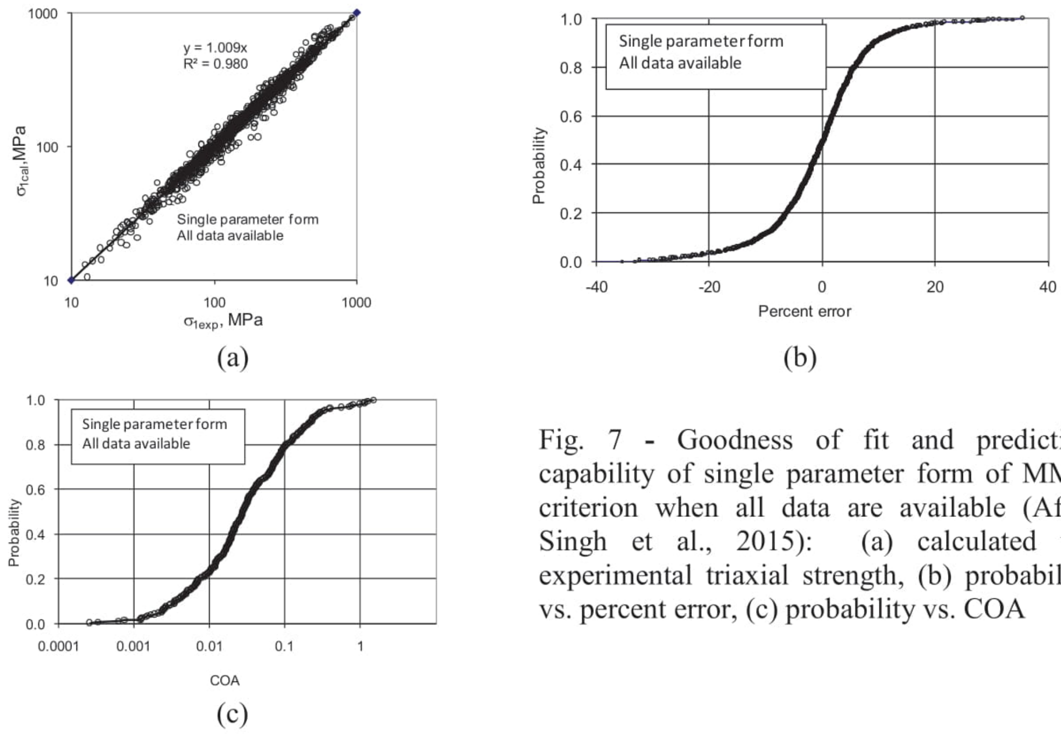


Fig. 7 - Goodness of fit and predictive capability of single parameter form of MMC criterion when all data are available (After Singh et al., 2015): (a) calculated vs. experimental triaxial strength, (b) probability vs. percent error, (c) probability vs. COA

The case (iii), where it is assumed that no triaxial test data is available to fit the parameter $\phi_{\beta0}$, is an extreme situation and may not arise. However in the field, many a times the only data available may be the UCS, especially during preliminary feasibility studies. An attempt has therefore been made to predict the strength even if no triaxial test data is available to assess $\phi_{\beta0}$. Using the entire data base compiled in this study, the term A (Eq. 14) was obtained for each data set and was plotted against σ_{crt} (Fig. 8). A correlation was found between term A and σ_{crt} with a R^2 value of 0.893 as follows:

$$A = -4.75\,(\sigma_{crt})^{-1.22} \tag{18}$$

Equations 15, 16 and 17 may now be used to obtain the parameter $\phi_{\beta0}$.

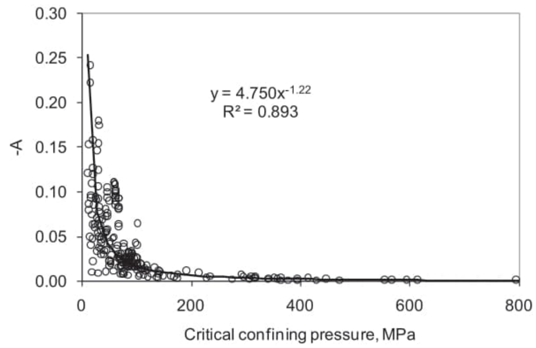
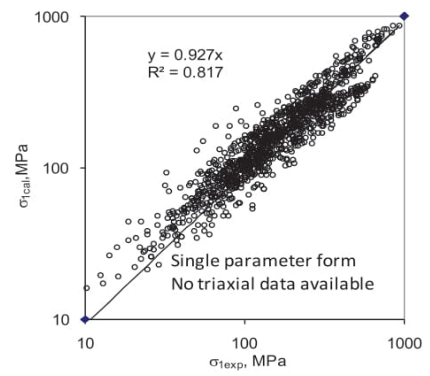


Fig 8 - Variation of term A with critical confining pressure (After Singh et al., 2015)

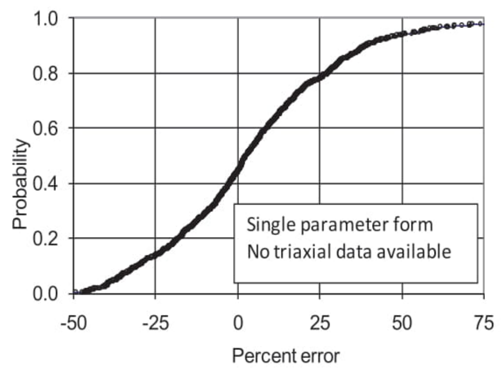
The values of $\phi_{\beta0}$ for the present data base were computed and the single parameter form of the criterion (Eq. 12 & 13) was used to estimate σ_{1cal} values. The plot of σ_{1cal} against σ_{1exp} is shown in Fig. 9a. The probability distributions of percent error and COA are given Figs. 9b & 9c respectively. The summary of analysis done for evaluation of goodness of fit and predictive capability is presented Table 3. It is seen that two parameters form has excellent predictive capabilities. Even the single parameter form also has extremely good predictive capabilities when all the triaxial test data points are used to fit the parameter $\phi_{\beta0}$ (case-i). If only two triaxial test data are used for fitting (case-ii), the predictive capability reduces. The results are still good. When no triaxial test data is available for fitting, the quality of prediction drops, however the prediction are still reasonable. The single parameter form may be used where rough estimates are required especially during feasibility studies and for relative comparison of different sites.

Table 3 - Summary of analysis for goodness of fit and predictive capability

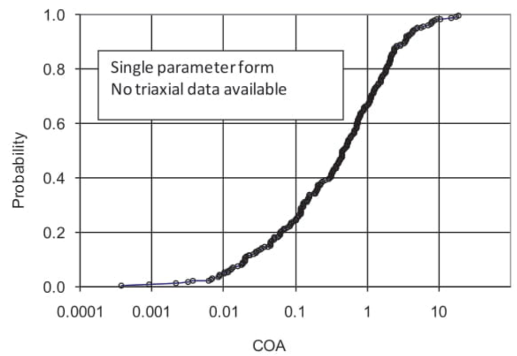
Condition	R^2	Probability of error to be within ±20%	AAREP	Probability of COA to be ≤0.1
Two parameter form	0.9954	0.9737	4.20%	0.9219
Single parameter form: case: i	0.980	0.9466	6.61%	0.7910
Single parameter form: case: ii	0.853	0.8364	12.38%	0.4922
Single parameter form: case: iii	0.817	0.5700	21.86%	0.2422



(a)



(b)



(c)

Fig. 9 - Goodness of fit and predictive capability of single parameter form of MMC criterion when no triaxial test data is available (After Singh et al., 2015): (a) calculated vs. experimental triaxial strength, (b) probability vs. percent error, (c) probability vs. COA

5. CONCLUDING REMARKS

A non-linear 'Modified Mohr-Coulomb (MMC) criterion' has been suggested to describe the triaxial strength behaviour of inherently anisotropic rocks. This simple criterion has been deduced by extending the conventional Mohr-Coulomb (MC) criterion and using the critical state concept of rocks. The general form of the criterion uses two parameters namely critical confining pressure σ_{crt} and friction angle $\phi_{\beta 0}$. The applicability of the criterion has been verified by applying it to a triaxial test data base comprising of test results on 38 rock types with total number of 255 UCS and 1141 triaxial tests. The performance has been evaluated statistically using the error indices regression coefficient R^2, percent error (pe), coefficient of accordance (COA) and average absolute relative error percentage (AAREP). The performance of the MMC criterion has been found to be excellent.

The analysis of the results from the data base indicates that, if sufficient tests data to fit the criterion parameters is not available, an approximate value of σ_{crt} may be taken nearly equal to $1.25\sigma_{cmax}$ without much compromise in accuracy of prediction. The only criterion parameter that will remain to be fitted will be $\phi_{\beta 0}$. The single parameter form of the MMC criterion thus obtained may be used with confidence to predict triaxial strength with minimum test data available on triaxial strength. It is also observed that reasonable estimates of the triaxial strength are possible, even in situations when no triaxial strength test data is available to fit the criterion parameter $\phi_{\beta 0}$.

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