Sequential Excavation of A Rectangular Underground Opening

Prabhat Kumar
Structural Engineering Division
Central Building Research Institute
Roorkee 247667 India
Tel. No. (01332) 72630, 72655
Fax (01332) 72272, 72543

ABSTRACT

The application of FEM in the solution of geotechnical engineering problems requires a strong recognition of the proper construction sequence. Efficient numerical strategies are available for this purpose. This paper presents an implementation and solution strategy of one such procedure. The blocky medium is replaced by an assembly of four noded rectangular finite elements for which all the necessary matrices can be derived in a close form. Several examples are solved, both analytically and numerically, to examine the performance of the sequential excavation simulation strategy. The outstanding issues are identified.

KEY WORDS:

Computer procedures; Finite element analysis; Geometrical nonlinearity; Rectangular opening; Sequential excavation; Shallow tunnel; Tunnels; Underground openings

1.0 INTRODUCTION

The excavation and construction sequence is of a particular relevence in the geotechnical engineering where important structures like embankments, tunnels, and pits are constructed through a gradual removal and/or addition of materials [Clough and Woodward, 1967; Duncan and Dunlop, 1969; Duncan and Clough, 1971]. The inelastic material behaviour further enhances the dependence of the final state of stress and deformation on the sequence in which the final structural configuration is realised.

The concept of incremental construction, which includes both addition and removal of material, was introduced by Clough and Woodward [1967] in which the incremental stress, strain and deformation due to a single construction step are computed and accumulated. The advancement in the finite element method added to the development of sequential construction procedures.

This paper presents recent developments and implementation of a procedure for finite element simulation of sequential construction. The structural domain is broken into an assembly of four noded rectangular finite elements for which all the property matrices can be written in a close form. Several examples are solved, both analytically and numerically, to examine the performance of this strategy. At this stage only the excavation strategy has been perfected. A subsequent publication will be devoted to the sequential construction.

2.0 MECHANICS OF EXCAVATION SIMULATION

Fig. 1 shows the balancing force system f which develops at the proposed excavation boundary. At the end of excavation, the excavation boundary has to be stress free. Therefore, the excavation may be simulated by analysing the excavated configuration under the action of balancing forces f and by adding the resulting stresses and deformations to the initial solution. More than one iteration may become necessary to arrive at the stress free excavation boundary.

3.0 FINITE ELEMENT SIMULATION OF SEQUENTIAL EXCAVATION

The finite element method is ideally suited to the simulation of sequential construction because the excavation corresponds to an elimination of a certain set of finite elements while construction requires generation of a set of finite elements. By choosing suitable shape and size of finite elements, it is possible to closely match the actual construction boundaries.

In a successful application of FEM to this problem, an accurate computation of the balancing forces is crucial. Some confusion was reported [Christian and Wong, 1973] which was subsequently corrected [Chandrasekaran and King, 1974]. However, the latter procedure which although satisfies the uniqueness condition [Ishihara, 1970], is strictly applicable only to the elastic materials. The uniqueness principle is the criterion of efficiency, accuracy and acceptability according to which there exists a unique solution independent of the sequence of cutting for a linear, time independent and elastic material. The uniqueness also includes step size independence. Besides the excavation surface must always be stress free Even though, the linear elastic materials may be involved, still the problem is nonlinear in which equilibrium iterations need to be performed.

Mana and Clough [1981] reported simulation of sequential excavation but did not achieve uniqueness. Desai and Sergand [1984] overcame the violation of the uniqueness principle from a nonzero nodal force vector acting on the removed nodes through a hybrid formulation. It adds considerable analytical complexity. Borja et al [1989] proposed a variational formulation based method which in conjunction with the concept of infinitesimal stiffness satisfies the uniqueness principle.

Ghaboussi and Pecknold [1984] demonstrated the validity of uniqueness principle by employing a virtual work formulation. This method is general and is applicable to nonlinear analysis [Ghaboussi et.al. 1983] Finally, Comodromos et al [1993] have proposed an algorithm for nullifying the nodal forces arising from excavated elements which in conjunction with a double pivoting strategy for the equilibrium solution process satisfies the uniqueness and stress free surface.

In the published literature, the sequential excavation and construction strategy has been described through simple examples which do not seem to pose any difficulty. However, examples presented towards the end of this paper demonstrate that intensive effort is required to arrive at an acceptable solution within reasonable number of iterations.

4.0 MATHEMATICAL FORMULATION

The equilibrium equations for the finite element model prior to excavation may be written in a partitioned form as,

$$\begin{bmatrix} K_{ee} & K_{eb} & 0 \\ K_{be} & K_{bb}^{c} + K_{bb}^{r} & K_{br} \\ 0 & K_{rb} & K_{rr} \end{bmatrix} \begin{Bmatrix} U_e \\ U_b \\ U_r \end{Bmatrix} = \begin{Bmatrix} Pe \\ P_b^{c} + P_b^{r} \\ P_r \end{Bmatrix} \qquad (1)$$

In eq. 1, the subscripts e, r and b refer to the excavated part, remaining part and the excavation boundary, respectively. K, U and P are stiffness, displacement and load, respectively. Similarly, the equilibrium equations for the excavated structure may be written in the partitioned form as,

$$\begin{bmatrix} K_{bb}^{r} & K_{br} \\ K_{rb} & K_{rr} \end{bmatrix} \begin{Bmatrix} U_b + \Delta U_b \\ U_r + U_r \end{Bmatrix} = \begin{Bmatrix} P_b^{r} \\ P_r \end{Bmatrix} \qquad (2)$$

In eq. 2, ΔU refers to the change in displacement due to excavation. U_b and U_r from eq. 2 may be eliminated with the help of eq. 1. The final relation is,

$$\begin{bmatrix} K_{bb}^{f} & K_{b}^{r} \\ K_{rb} & K_{rr} \end{bmatrix} \begin{Bmatrix} \Delta U_b \\ \Delta U_r \end{Bmatrix} = \begin{Bmatrix} P_b^{r} - I_b^{r} \\ 0 \end{Bmatrix} \qquad (3)$$

where

$$I_b^{r} = K_{bb}^{r} \quad U_b + K_{br} \quad U_r = -(K_{be} \quad U_e + K_{bb}e \quad U_b)$$

Eq. 3 demonstrates how the equivalent nodal forces corresponding to the reversed balancing forces across the excavation surface must be computed in order to preserve step size independence. This procedure may be progressively repeated till arriving at the final excavated configuration.

Several strategies for the computation of I_b^r and I_b^e are available [Ghaboussi and Pecknold, 1984]. In thepresent study the excavated elements are deactivated before assembling the incremental equilibrium equations. The equivalent nodal forces are picked from the known geometry of the excavated surface. These forces are then reapplied to the excavated configuration to obtain incremental solution. The displacements U, strains ε and stress σ are updated using the well known relations,

$$U_n = U_{n-1} + \Delta U_n \quad \text{(similarly for } \varepsilon \text{ and } \sigma) \qquad (4)$$

It may be possible to annihilate the equivalent nodal forces either in one application or in a number of load steps. An application of this strategy can be found in [Kumar, 1996].

5.0 FINITE ELEMENT IMPLEMENTATION

The previously described procedure has been implemented in a computer program. This computer program although is designed to handle both excavation and construction steps which may be executed in any order. However, only the excavation part has been tried so far. It includes the following subroutines.

MAIN	Total memory is allocated and title of the problem is read and printed.
CHIEF	Driver routine in which values of various control variables are read and the memory is partitioned into arrays.
INPUT	Read node coordinates, element connectivity, material properties and element properties. Excavation stage data including the list of element excavated in a stage is read.
STAGE	This subroutine calls DACO to read element data for construction stages and EXCAV for an excavation stage.
DACO	It is stipulated that the additional nodes needed in construction may be manipulated from the nodes which have been excavated previously by assigning new node coordinates. Thus, no new nodes need be added to allow construction.
EXCAV	This subroutine identifies nodes which lie at the current interface of excavated and unexcavated parts.Also,the excavated nodes are identified.
ARRANG	Rearrange the information generated by EXCAV for subsequent use.
FIXN	Read the data of nodes which are to have fixity to simulate supports.
DATPRI	Print all the problem data and other information in a tabular format.
BOUND	Read the prescribed non-zero displacements at the supports if present.

LINKIN	Assign global degrees of freedom numbers to various nodes of structure.
COLHT	Compute the column hights for the skyline storage of the global matrix.
ADRES	Compute address of the diagonal coefficients of global stiffness matrix
MODIFY	The diagonal coefficient address array is modified to accomodate DOF's which are constrained.
STRSTN	This subroutine directs the major finite element computational effort.
ASMSKY	Call BLOCK and ASMB through which the global stiffness matrix is assembled in a one-dimensional format.
GLOAD	Compute the gravity load vector for each active element and load it in the global load vector through ASMRH.
SKYFAC	This subroutine factorizes the skyline stored global stiffness matrix.
SKYSOL	This subroutine does forward reduction and back substitution operations
SIMU	Simulate the excavation or construction through SIMEX or SIMCO routines
SIMCO	Release the necessary nodes to be used in construction. The nodes are assigned new coordinates and elements are given new connectivity as input through subroutine DACO.
SIMEX	Fix the nodes to be excavated which are identified by subroutine EXCAV.
RESIDU	Compute stresses and residual nodal forces. Also, generate balancing force vector as per eq. 3.
UPDATE	Incrementally update the displacements (eq. 4). Also, assemble reaction vector.

CONVER Check the convergence of the newly computed incremental solution with a known tolerence factor.

OUTPUT The displacements, reactions and stresses at the sampling points are printed in a tabular form.

These subroutines are organised in a computer program as shown in Fig. 2. Two add-on modules identified as 1 and 2 carry out the computations necessary for sequential procedure. Without these modules, the computer program performs a typical finite element analysis. These modules are activated when the control variable NSTAGE is greater than zero. The module 1 identifies the nodes to be excavated and defines the interface between the excavated and remaining portions. The module 2 computes the balancing forces and applies these at the previously computed interface nodes. The add-on character of these modules ensures that any FEM analysis computer program can be converted to have the facility of sequential procedure without extensive revisions. The only additional input which becomes necessary to implement the sequential excavation strategy is the number of stages, maximum number of elements involved in any stage, stage description and stagewise element identification. Everything else is automatically computed. Also, all computations are done in double precision.

The Fig.2 showing computer program contains only four-noded rectangular elements because the properties of these elements can be worked out in a close form leading to programming convenience.

Solution Example 1

Clough and Mana [1981] solved a one-dimensional excavation problem (Fig. 3) in which the upper half of the material is to be excavated. An exact solution of this problem is as follows,

$$\sigma_{yy} = \frac{E}{1+\nu}\varepsilon_{yy} + \frac{\nu E}{(1+\nu)(1-2\nu)}\theta$$

where $\theta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$

In plane strain conditions, $\varepsilon_{zz} = 0.0$. Also by virtue of boundary condition,

$\varepsilon_{xx} = 0.0$, therefore,

$$\sigma_{yy} = \left[\frac{E}{1+\nu} + \frac{\nu E}{(1+\nu)(1-2\nu)}\right]\varepsilon_{yy}$$

Since $\sigma_{yy} = 20.0\ t/m^2$; $\varepsilon_{yy} = 1.8 * 10^{-3}$ and $u_y = 3.6$ cm. Here s and e are the stress and strain. U_y is the vertical displacement at the excavation surface. If the sequential excavation strategy is accurate, then the excavation surface must show uniform displacement irrespective of how the excavation is achieved.

The FEM model of this problem contains 45 nodes and 32 finite elements. An analysis of the unexcavated structure yielded a vertical downward displacement of 0.054 meters at the excavation surface. A similar analysis of the fully excavated structure yielded a vertical downward displacement of 0.018 meters. This suggests that a recovery of displacement due to excavation of amount 0.036 meters took place, which varifies the analytical result.

The balancing forces are computed analytically in Fig. 4 for one step excavation. A similar analysis can be performed for a multi-step excavation. In the finite element analysis, the balancing forces were obtained exactly as in Fig. 4 and the solution converged in just one iteration. The results of one, two and four step analysis were identical and exactly matched with the analytical predictions. This verifies the uniqueness as well as step size independence criteria of the sequential excavation simulation procedure. The stress changes in a particular element at various stages of excavation are shown in Fig. 5. In this example the elements remain under a state of bi-axial compression before and after the excavation. The favourable convergence characteristics may be attributed to this feature which will be further explored in the subsequent example. The convergence criterion used in this study is as follows,

$$\frac{\sqrt{\Sigma\ \delta_i^2}}{\sqrt{\Sigma\ \delta_T^2}} \leq \quad TOL \qquad (7)$$

Where d are displacements, subscripts i and T stand for incremental and total respectively and the summation is done over all nodes. TOL is the user specified convergence tolerence. In this study TOL = 0.001 is used.

Solution example 2.

Excavation of a rectangular opening.

The geometry of the rectangular opening which is to be excavated and its FEM model are shown in Fig. 6 & 7, respectively. Because of symmetry, only half of the problem need to be solved. Its FEM model contains 273 nodes and 240 elements. The gravity loading is applied.

The importance of sequential excavation process is demonstrated by following the stress in element numbers 6 and 61 (Fig. 8) in a three stage excavation process. In Fig. 8, an arrow pointing towards and away from the centre denote compression and tension, respectively. Fig. 8 shows that a complete or partial stress reversal take place due to excavation. Also, the stresses after the first stage of excavation are higher than those at the end of complete excavation. This may be due to the stress redistribution which is initiated by the excavation and increases with the size of opening. The large horizontal tensile stress at the top of the opening develop due to beam action The vertical tensile stresses in element 61 indicate that the roof is in need of external support. However, good quality rock may be able to sustain small amount of tensile stress eventhough the existing rock engineering practice may not allow this. Therefore, the sequential excavation analysis is realistic as well as important from the safety point of view.

Following the procedure outlined by Ghaboussi and Pecknold [1984], it is possible to compute the balancing forces. These are shown in Fig. 9 for a single stage excavation. The numerical values were identical with these. Similar computations can be done for intermediate stages of a multi-stage process of excavation. In a sigle stage excavation, the solution converged to a tolerance of 0.001 after 28 iterations. The progress of convergence is studied in Table 1. The exact values in Table 1 were obtained from a study in which the final configuration was analysed.

As in example 1, the balancing forces are exactly computed but a one-cycle convergence is not achieved. A study of Table 1 shows that the solution converges very fast for about first 10 iterations,then slows down considerably All the values in the converged solution are acceptable except the vertical stress in element 61 which although constantly decreased but could not decrease fast enough and far enough to become tensile. Some evidence of very minor oscillations can also be seen in the convergence.

Also shown towards the bottom of Table 1 are the converged solutions at the end of two step and three step excavation procedures. Each excavation step consists of removal of three and two layers at a time, respectively. The numerical stress history of elements 6 and 61 is shown in Fig. 10.

So far in the solutions of two examples, equal sized steps have been employed. But the sequential excavation strategy is not restricted to it. This is shown by excavating the rectangular opening under consideration in three steps such that the steps consists of removal of one, two and three layers, respectively. The results of converged solution are shown in the last row of Table 1.

Table 2 shows the number of iterations required in previously mentioned excava tion schemes. As can be seen, the computational effort increases substantially with the number of excavation stages. It may be possible to improve the numerical performance of the sequential excavation strategy by employing higher order finite elements and by using infinite elements for modelling of unbounded analysis domain. This work is in progress and shall be reported at an appropriate time.

The sequential excavation study can also be performed to determine a fvourable sequence of excavation by analysing various possible sequences before actually starting the excavation on the site. In general, the sequence of excavation may depend upon the rock quality, size of opening, method of excavation and the intensity of initial stress field.

The results obtained in the solution of rectangular opening excavation suggest that the uniqueness principle and the step size independence has been achieved in the present strategy and its implementation.

6.0 CONCLUDING REMARKS

Various strategies for simulation of sequential underground excavation are required to satisfy the uniqueness principle and step size independence. One such strategy was adopted and implemented in a FEM analysis computer program in the form of two add-on modules. The solution of sample problems reveal that the balancing forces are exactly computed, however, one-cycle convergence is achieved only if stress reversals due to excavation do not take place.

Also, the bi-axaial stress reversals may pose convergence difficulties. The convergence tolerance of 0.001 has been found adequate in the sample problems. The implementation meets the abovementioned criteria. It is appropriate to mention that such a strategy is applicable to excavation of slopes and construction of embankments.

The computations show that in a multi-stage excavation the stresses at the end of a partial stage are more severe than at the end of complete excavation. This finding is of practical importance and due consideration must be given to it in the support design.

7.0 ACKNOWLEDGEMENT

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8.0 REFERENCES

1. Borja, R.I., Lee,S.R. and Seed, H.B. (1989), "Numerical simulation of excavation in elasto-plastic soils", Int. J. Num. and Anal. Methods in Geom., 13: 231-249.

2. Chandrasekaran, V.S. and King, G.J.W. (1974), "Simulation of excavation using finite elements", J. Geotech. Eng. (ASCE), 100: 1086-1089.

3. Christian, J.T. and Wong, I.H. (1973), "Errors in simulation of excavation in elastic media by finite elements", Soils and Found., 13: 1-10.

4. Clough, G.W. and Woodward, R.J. (1967), "Analysis of embankment stresses and deformations", J. Soil Mech. Found. Eng. (ASCE), 93: 529-549.

5. Comodromos, E., Hatzigogos, Th. and Pitilakis, K. (1993), "Multi-stage finite element algorithm for excavation in elastoplastic soils", Comp and Struct., 46: 289-298.

6. Desai, C.S. and Sergand, S. (1984), "Hybrid FE procedures for soil-structure interaction", J. Geotech. Eng. (ASCE), 110: 473-486.

7. Duncan, J.M. and Dunlop, P. (1969), "Slopes in stiff fissured clays", J. Soil Mech. Found. Eng. (ASCE), 95: 467-492.

8. Duncan, J.M. and Clough, G.W. (1971), "Finite element analysis of Port Allan Lock", J. Soil Mech. Found. Eng. (ASCE),97: 1053-1066.

9. Ghaboussi, J. and Hansmire, W.H., Parker, H.W. and Kim, K.J. (1983), "Finite element simulation of tunnelling over subways", J. Geotechnical Engng.(ASCE) 109: 318-334.

10. Ghaboussi, J. and Pecknold, D.A. (1984), "Incremental finite element analysis of geometrically altered structures", Int. J. Num. Methods in Eng.,20: 2051-2064.

11. Ishihara, K. (1970), "Relations between process of cutting and uniqueness of solutions", Soils and Found., 10: 50-65.

12. Kumar, P. (1996), "Sequential excavation analysis of underground openings", IGC-96, Indian Geotechnical Society, Anna University, Madras.

13. Mana, A.I. and Clough, C.W. (1981), "Prediction of movements for braced cuts in clays", J. Geotech. Eng. (ASCE), 107: 759-777.

TABLE 1

Study of convergence in sequential excavation of rectangular opening

Iter. No.	Conver. factor (*0.01)	Defl. at node 1	Defl. at node 79 (* 0.1)	stress in element 6 hor.	6 ver.	61 hor.	61 ver.	136 hor.	136 ver.
exact	0.100	1.111	1.117	0.88	-1.15	4.25	0.485	-7.80	-40.69
1	9.03	0.940	0.902	0.31	-1.06	-3.24	-6.9	-5.09	-23.36
2	3.77	0.964	0.918	0.22	-1.05	-1.9	-8.18	-5.78	-29.17
3	1.27	0.972	0.932	0.37	-1.06	-1.4	-7.23	-6.24	-30.77
4	1.00	0.984	0.95	0.40	-1.07	-0.71	-7.08	-6.72	-32.82
5	0.54	0.994	0.96	0.455	-1.08	-0.25	-6.58	-7.03	-34.00
6	0.59	1.00	0.972	0.491	-1.08	0.176	-6.20	-7.26	-35.02
7	0.49	1.012	0.983	0.530	-1.09	0.54	-5.80	-7.40	-35.80
8	0.47	1.02	0.994	0.56	-1.10	0.87	-5.40	-7.50	-36.40
9	0.426	1.027	0.100	0.59	-1.10	1.16	-5.00	-7.54	-36.93
10	0.395	1.034	1.014	0.62	-1.1	1.435	-4.70	-7.57	-37.36
12	0.336	1.050	1.030	0.67	-1.12	1.92	-4.03	-7.6	-38.03
14	0.290	1.060	1.045	0.71	-1.13	2.35	-3.48	-7.6	-38.53
16	0.245	1.066	1.058	0.75	-1.134	2.72	-2.99	-7.58	-38.91
18	0.209	1.074	1.070	0.79	-1.14	3.05	-2.57	-7.56	-39.21
0	0.180	1.080	1.078	0.82	-1.15	3.33	-2.21	-7.54	-39.45
22	0.154	1.087	1.087	0.85	-1.15	3.58	-1.89	-7.52	-39.64
24	0.132	1.092	1.094	0.87	-1.154	3.80	-1.62	-7.5	-39.8
26	0.113	1.096	1.10	0.893	-1.157	3.98	-1.39	-7.49	-39.94
28	0.098	1.1	1.105	0.912	-1.160	4.146	-1.187	-7.47	-40.05
AA	0.096	1.106	1.115	0.983	-1.13	4.70	-0.719	-8.13	-39.84
BB	0.099	1.108	1.12	1.033	-1.20	4.89	-0.622	-7.01	-39.26
CC	0.093	1.107	1.116	0.994	-1.116	4.75	-0.656	-8.08	-39.82

NOTE: *AA stands for two stage excavation sequence.*
BB stands for three stage (equal) excavation sequence.
CC stands for three stage (unequal) excavation sequence.
All figures in this table have been rounded off.

TABLE 2

Convergence properties of problem 2 with tolerence = 0.001.

Construction Sequence	Number of iterations at the end of *1st stage*	*2nd stage*	*3rd stage*	Total
One stage	28	-	-	28
Two stage	27	13	-	40
Three stage (equal)	25	11	10	46
Three stage (unequal)	27	11	13	51

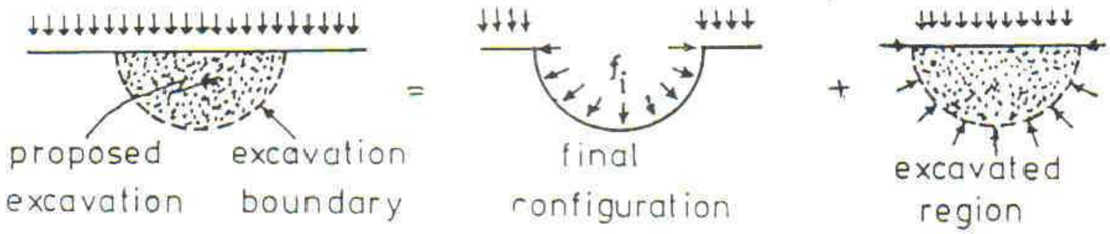


Fig. 1 One stage excavation example

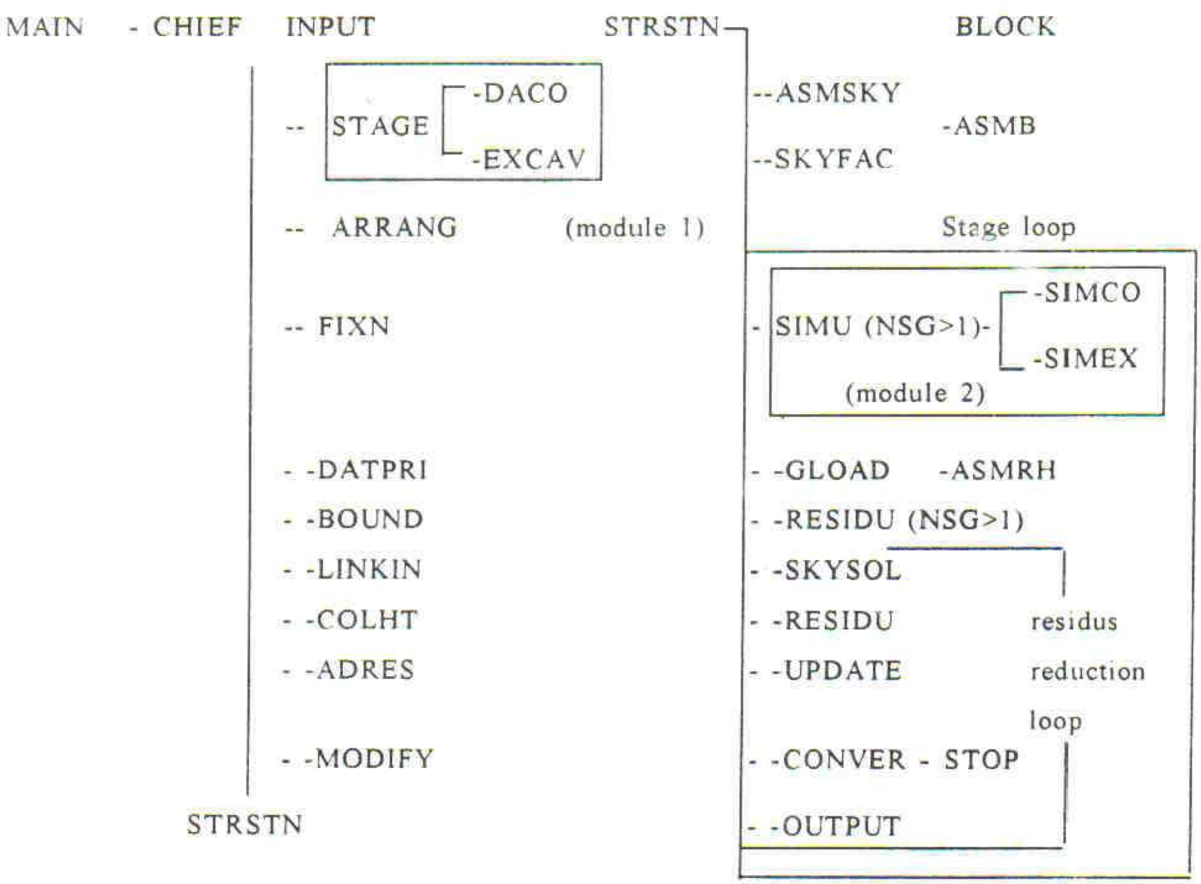


Fig. 2 Flow chart of computer program for sequential construction simulation

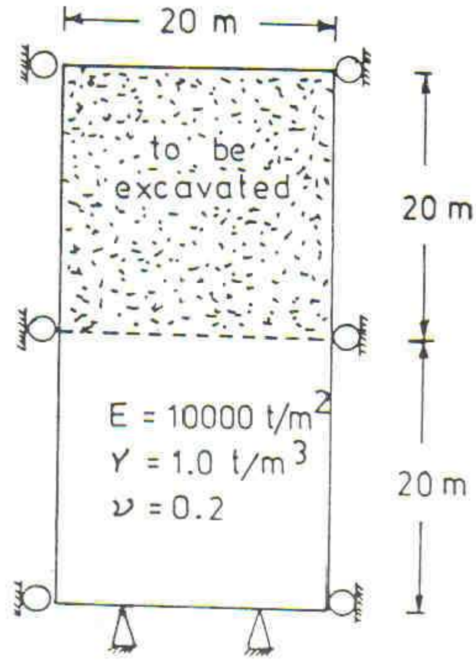


Fig. 3 Clogh and Mana: problem geometry.

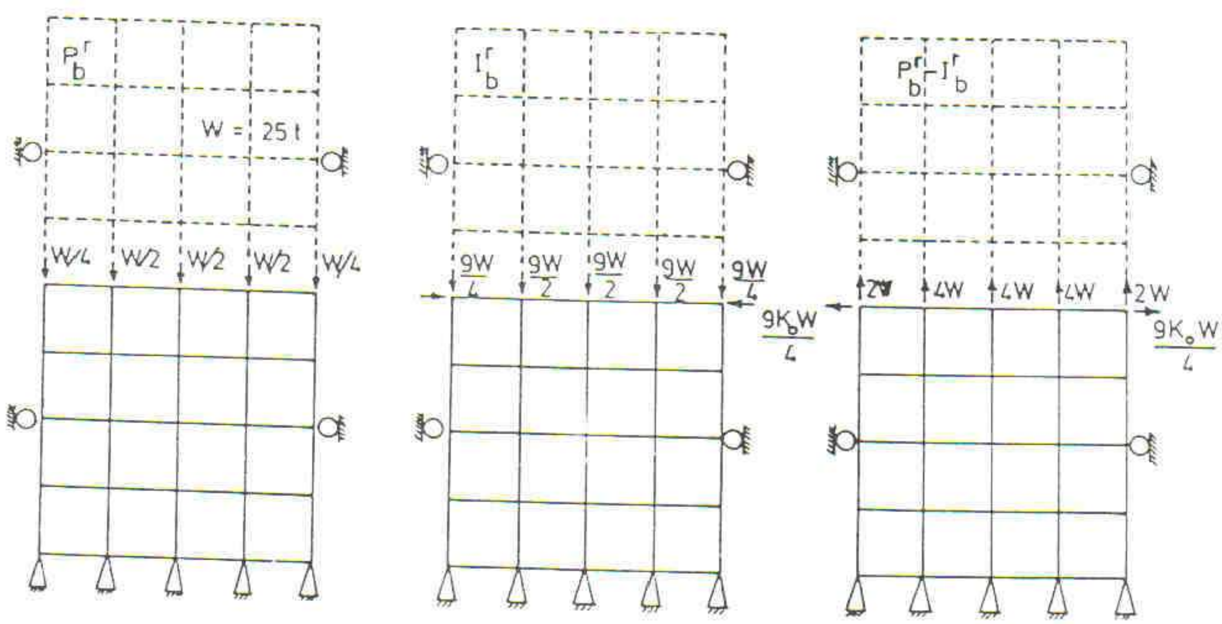
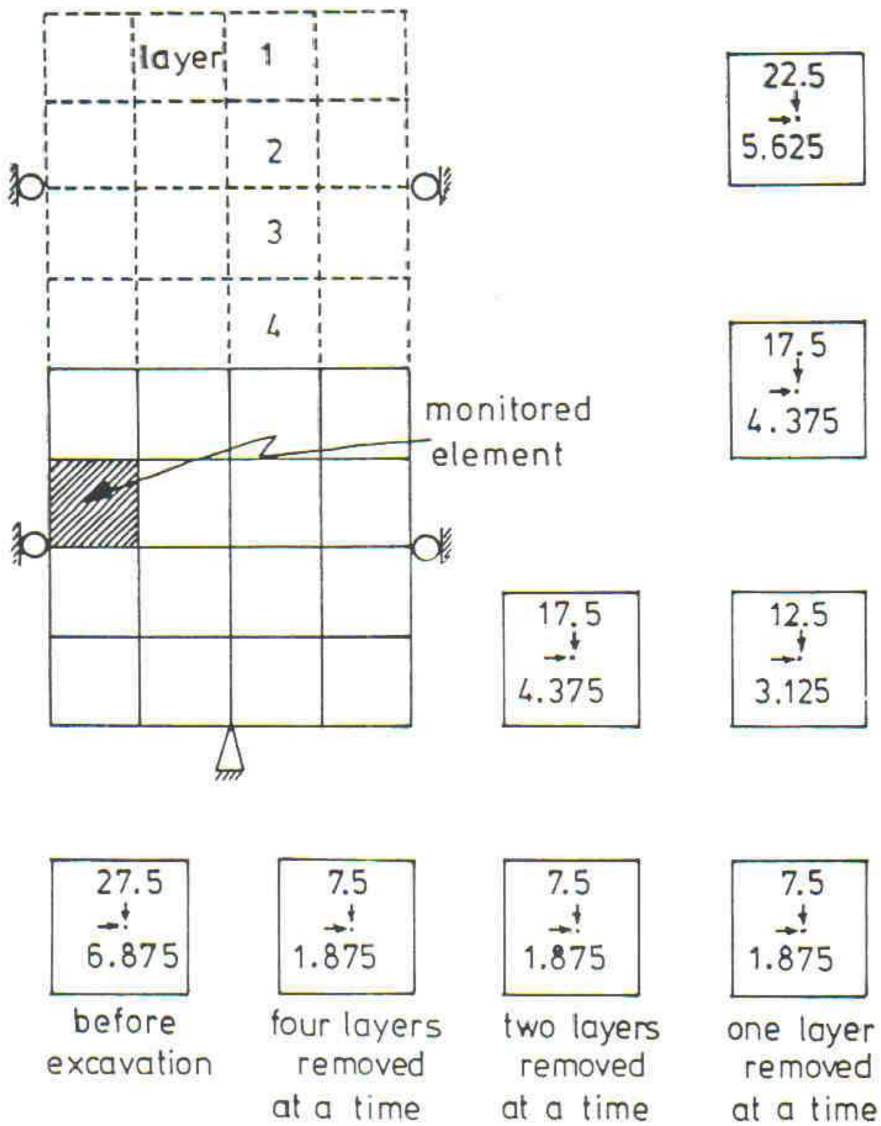


Fig. 4 Analytical computation of balancing forces.



ig. 5 **Element stress changes with stepwise excavation.**

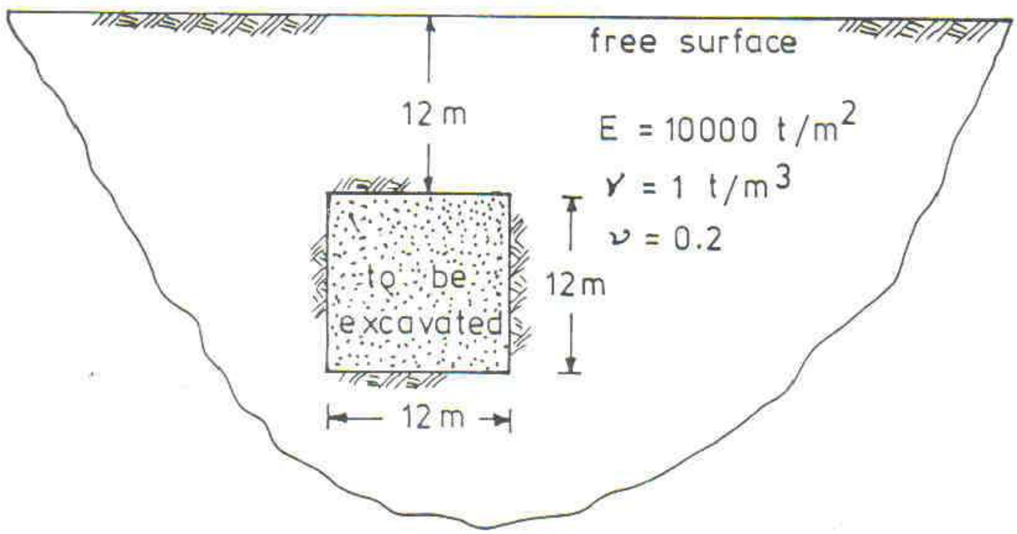


Fig. 6 Geometry of rectangular opening problem.

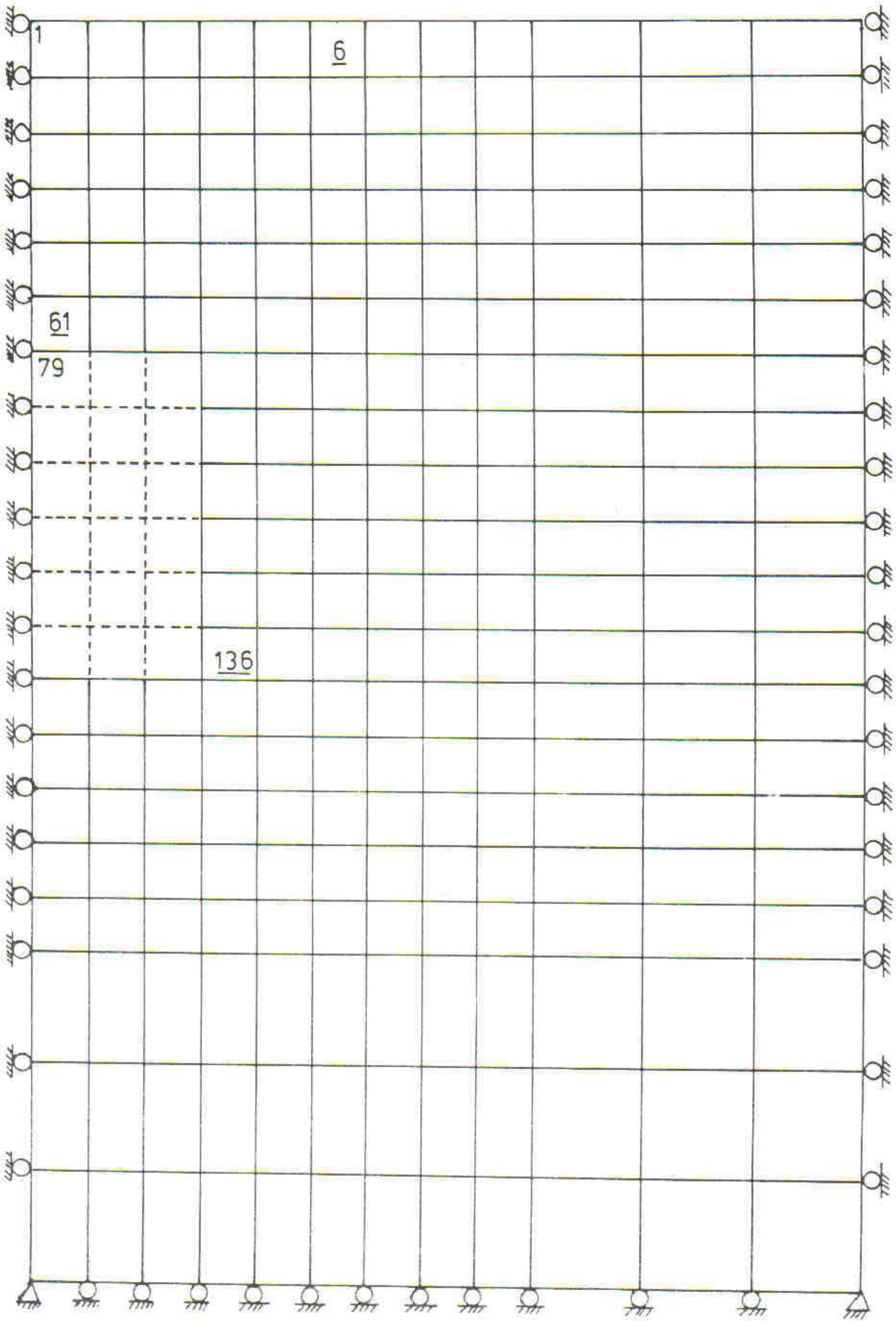


Fig. 7 FEM model of rectangular opening problem.

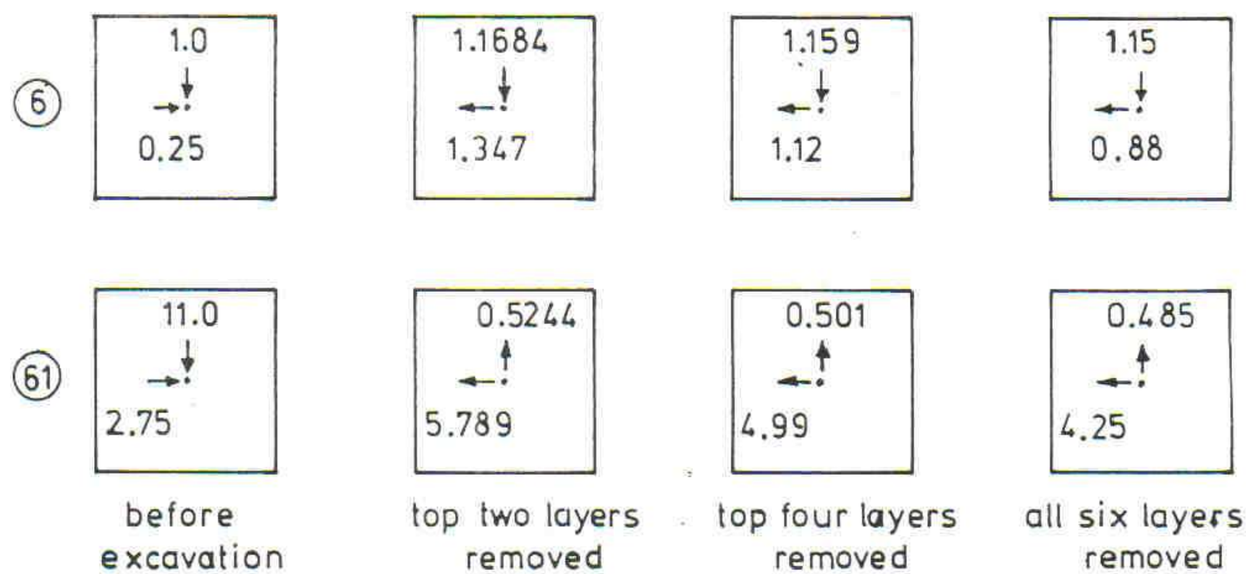
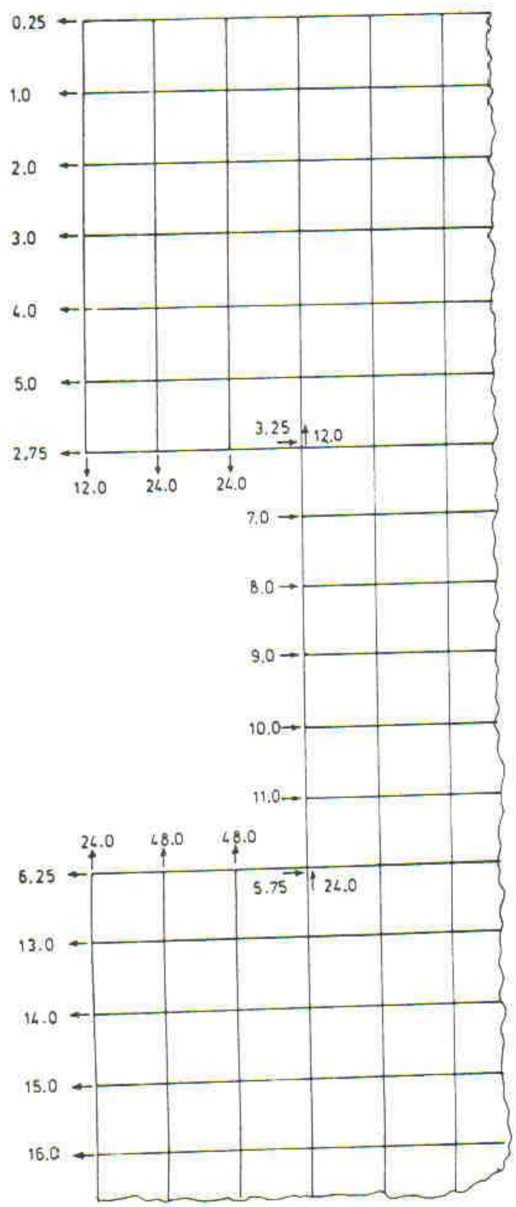


Fig. 8 Stress history of elements 6 and 61 during three stage excavation.



9 **Balancing forces corresponding to single stage excavation.**

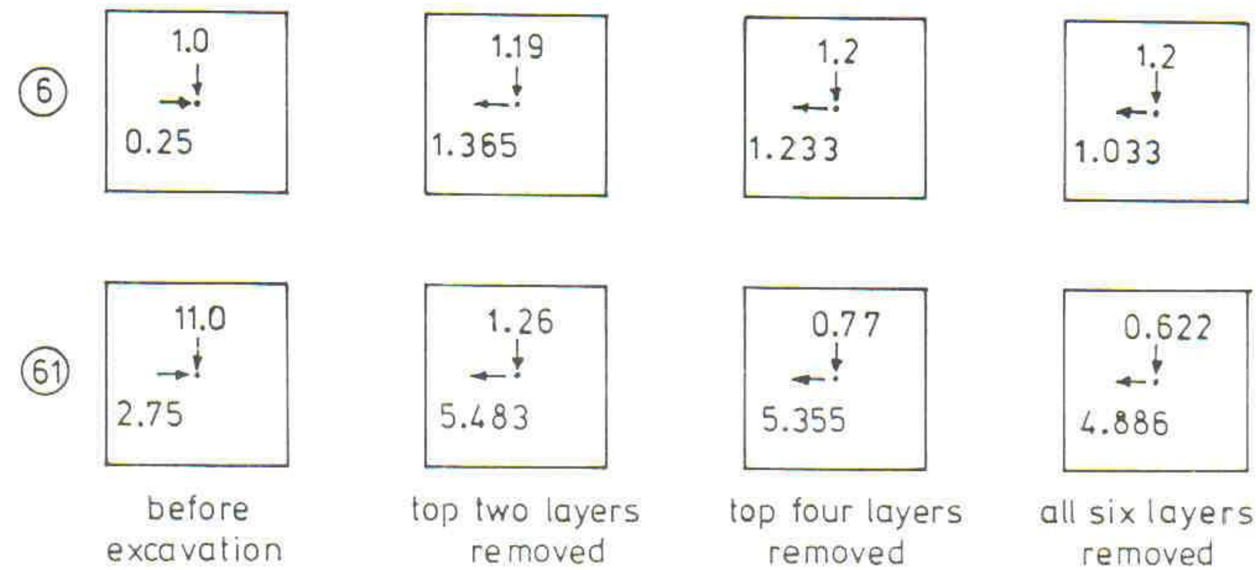


Fig. 10 Numerical stress history of elements 6 and 61 during three stage excavation.