Non-Linear Analysis of Jointed Rock Using Equivalent Continuum Approach

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ABSTRACT

This paper deals with finite element modelling of jointed rock as an equivalent continuum whose properties represent the material properties of the jointed rock. The material properties of the equivalent continuum are represented by a set of empirical relationship which express the properties of the jointed medium as a function of joint factor and the properties of the intact rock. The reliability of the analysis depends upon the joint factor which is a function of the joint orientation, joint frequency and joint strength. So, for a particular jointed medium knowing the intact rock properties and by estimating the joint factor, the properties of the jointed rock can be determined. The result obtained by the above analysis have been compared with the experimental results for three different rock materials for both intact and jointed medium. The results are in good agreement even for single and multiple joints and this can be easily extended to materials with many joints. The accuracy of the results for a discontinuous system depends on how well the joint factor is estimated for that particular case. Attempts are underway to apply it to real discontinuous systems.

1.0 INTRODUCTION

Rock masses generally contain pre-existing discontinuities which influence the strength of intact rock. These discontinuities are mainly due to geological, lithological, physical, mechanical and environmental factors. The presence of joints and faults renders the rockmass anistropic in its response to the applied stress field. These discontinuities weaken the rock masses to an extent which depends very much on the size of engineering structure in relation to discontinuity spacing. Although the properties of the intact rock between the discontinuities and the properties of the joints themselves can be determined in the laboratory, the properties of the rock mass are not amenable to direct physical measurements. For determining the rock mass properties indirectly, a theory is needed to be established and tested in some independent way.

A number of experimental studies have been conducted both in-situ and laboratory to understand the behaviour of natural as well as artificial joints. In-situ tests have been carried out to study the effect of size on rock mass compressive strength. The results of these investigations clearly show the reduction of mass strength and modulus with size up to a certain size at which change becomes insignificant. It is important to note that these relations are highly site-dependant, since the scale effect is primarily governed by the fracture network. In these invesigations no attempt has been made to map the joint network. Artificial joints have been studied mainly as they have advantage in their reproducibility. The anistropic strength behaviour of shales, slates and phylites has been investigated by a large number of investigators. Laboratory studies show that many different failure modes are possible in jointed rock and that the internal distribution of stresses within a jointed rock mass can be highly complex. Due to large expense and time involved in experimental studies, coupled with the demand of highly accurate techniques for measurements, a number of investigators attempted to study the behaviour of joints using analytical models. A number of strength criteria were proposed by various investigators which were bound by certain assumptions and limitations in thier application.

The numerical approach of treating the rock mass with equivalent material properties for obtaining the overall response of rock has been strongly advocated in recent years. In equivalent continuum approach the properties of jointed rock masses are expressed as a function of the intact rock properties and the properties of rock joints. In this paper, the jointed rock has been represented as a equivalent continuum and analysed by finite element method. The expressions given by Ramamurthy (1994) have been used to represent the jointed rock as an equivalent continuum. Non-linear elastic (confining stress dependent) finite element model following an hyperbolic relationship suggested by Janbu (1963) is used for the intact rock properties. The results have been compared with the laboratory stress-strain curves of the jointed rock for different values of the inclination of the discontinuities and for different materials.

2.0 CONSTITUTIVE BEHAVIOUR OF JOINTED SYSTEMS

The properties of the jointed rock masses are of concern when compared to that of the intact rocks or rock joints. The most popular failure criterion for rock discontinuities was proposed by Barton and Choubey (1977). In this model the base friction angle of the rock, unconfined compressive strength, joint roughness parameter (JRC), and the level of normal stress at which the shear test was conducted were used to predict the shear strength of jointed rock. Joint Roughness Coefficients are meant to be chosen by matching the standard profiles with the profile from the joint surface for which shear strength has to be estimated.

The first attempt to predict the properties of jointed rock masses for a known joint fabric based on the properties of intact rocks and rock joints was made by Hoek and Brown (1980) who proposed an empirical failure criterion to estimate rock mass strength. Even though they looked into the anistropy of rock mass strength, the Hoek and Brown criterion for rock mass failure used in practice is based on isotropy. This criterion includes three parameters:

(i) the uniaxial compressive strength of intact rock,
(ii) m, a parameter which depends on the rock type and,
(iii) s, a parameter whose value is determined according to the Rock Mass Rating (RMR).

The value of these two parameters m and s are analogous to angle of internal friction and cohesive strength in the conventional failure criterion. The values of these constants were given based on the extensive laboratory and field data. The RMR system by Bieniawski (1968) includes both the joint spacing and RQD, which are the measures of one-dimensional joint intensity where as joint size is not included as a parameter. However, joint size plays a very important role in rock mass strength. For a rock mass, one-dimensional joint intensity changes with the direction and thus can have infinitely many values. Kulatilake et al,(1997) showed that by knowing the one-dimensional intensity along the mean vector direction for each joint set, it is possible to calculate the one dimensional intensity in any direction for a jointed rock mass. Roy et al.(1995) also suggested expression for unconfined compressive strength of jointed rock in terms of rock mass rating (RMR) and the intact rock strength. Based on extensive laboratory test results, they have presented empirical relationships for elastic modulus and unconfined compressive strength of the jointed rock mass in terms of joint factor and intact rock properties.

Finite element models based on equivalent continuum approach are as follows:

In the multi-laminate model by Zeinkiewicz and Pande (1970) the plastic strain vector is expressed in terms of contribution of intact rock to the plastic strain rate and the contribution of k-th joint set.

In the Gerrard (1982) model, the compliance of an element of the equivalent continuum is expressed as the sum of compliance of the intact rock and that of the individual joint sets. In the computation of the equivalent continuum compliance for an element, it is assumed that the element stress and strain can be adequately described in terms of an average stress vector and strain vector defined at the element centroid.

Holland et al.(1997) gave a comparison of discontinuum model using UDEC with equivalent continuum model based on Hoek and Brown emprical criteria. The equivalent friction angle and cohesion of the rock mass based on RMR recommendations and Hoek and Brown criteria are generally conservative than the results predicted by the discontinuum models.

In the present paper an attempt is made to numerically model the multiple jointed systems using equivalent continuum approach. The advantage of this approach is that it is simpler to model the highly jointed rock masses using the finite element approach by representing the rock mass as equivalent continuum.

3.0 STATEMENT OF THE PROBLEM

The stress-strain behaviour of rocks over a wide range of stress field is non-linear and dependent upon confining pressure. This has brought down the credibility of the deformation modulus obtained from uniaxial tests. The intact rock behaviour has been modelled using the following relationship given by Janbu.

$$E_i = KP_a \left(\frac{\sigma_3}{P_a}\right)^n \qquad [1]$$

Where, E_i is Young's modulus of the intact rock, K modulus number, n is the modulus exponent, σ_3 confining pressure and P_a atmospheric pressure.

A series of tri-axial compression tests have been conducted on the block jointed systems using cubic blocks with different joint orientations and unjointed joint material [Brown and Trollope (1970)]. The mechanical behaviour of the most simple block jointed system is markedly different from the unjointed specimen and using the computer analysis a power law was fitted to the test results. The difficulty involved in the application of the power law to the practical problems is that it requires appropriate strength parameters for each rock mass to be determined experimentally. Triaxial compression testes were conducted on prismatic samples in which parallelopiped and hexagonal blocks were used to produce intermittent joint planes and to simulate more complex and real practical problem [Brown (1970)]. The exact solution to the stress distribution within the samples is most difficult to determine experimentally and quantitative application of model test results to practical situations is difficult. These model test results are very useful to indicate possible modes of failure and applicability of various strength theories. Expreiments were performed on gypsum models of a rock mass having joint sets with different spacings and different orientations Eienstein and Hirschfield (1973). They arrived at generalised Mohr plot to represent the behaviour of the model rock mass with various joint configurations. The systematic behaviour predicted in the plot can be used for the practical application of the results; given the knowledge of the intact material, the joint configuration and the stress state.

Extensive laboratory testing of intact and jointed specimens [Arora (1987), Yaji (1984), Ramamurthy (1996)] of different grades of Plaster of Paris, Sandstone and Granite in uniaxial and triaxial compression revealed that the important factors which influence the strength and modulus values of the jointed rock are:

(i) joint frequency J_n (no. of joints per meter depth),
(ii) joint orientation, with respect to major principal stress direction and,
(iii) joint strength.

The Joint factor is given as

$$J_f = \frac{J_n}{nr} \qquad [2]$$

n is the inclination parameter obtained by taking the ratio of log (strength reduction) at $\beta = 90$ to log (strength reduction) at the desired value of β. This inclination parameter is independent of joint frequency. The values of n are given for various orientation angles. The joint strength parameter r depends on the uniaxial compressive strength of intact rock and the values are given in the form of table based on extensive laboratory testing. The values of r are given for both unfilled joints and filled joints with gouge material.

The jointed rock has been modelled using the empirical relationships based on the joint factor. These relationships have been established by Roy (1995) based on the test results of Arora (1987), Yaji (1984), Brown and Trollope (1970), Einstein and Hirschifield (1973) and Ramamurthy (1994). The empirical relations for unconfined compressive strength of the joint specimen are as follows :

For upper bound,

$$\sigma_{cr} = \frac{\sigma_{cj}}{\sigma_{ci}} = \exp(-0.004\ J_f) \qquad [3]$$

For average limit,

$$\sigma_{cr} = \frac{\sigma_{cj}}{\sigma_{ci}} = \exp(-0.008\ J_f) \qquad [4]$$

For lower bound

$$\sigma_{cr} = \frac{\sigma_{cj}}{\sigma_{ci}} = \exp(-0.01\ J_f) \qquad [5]$$

The empirical relations for the modulus reduction factor (MRF) are as follows:

For upper bound,

$$E_r = \frac{E_j}{E_i} = \exp\ (-0.70\ \times\ 10^{-2}\ J_f) \qquad [6]$$

For average limit,

$$E_r = \frac{E_j}{E_i} = \exp\ (-1.15\ \times\ 10^{-2}\ J_f) \qquad [7]$$

For lower bound,

$$E_r = \frac{E_j}{E_i} = \exp\ (-1.80\ \times\ 10^{-2}\ J_f) \qquad [8]$$

Where σ_{ci} is the uniaxial commpressive strength of intact rock, σ_{cj} uniaxial compressive strength of jointed rock, E_j is the modulus of deformation of jointed rock and J_f is the joint factor. The effect of joint factor on the modulus of jointed rocks and on the unconfined compressive strength is as shown in Figs. 1 and 2 respectively. The above equations hold good for number of rocks and rock like material covering a wide range of σ_{ci} and E_i. Any one of the equations can be chosen based on the factor of safety to be adopted for the particular field problem. The upper bound limit equation can be used for higher factor of safety.

4.0 METHODOLOGY

In the present approach the jointed rock has been analysed by finite element method using the equivalent continuum approach. The modulus of intact rock has been obtained by using the Eq.1. The modulus of deformation of the jointed rock is obtained from the modulus of intact rock by evaluating the joint factor of the jointed rock using Eqs.2 and 7. Finite Element code for the equivalent continuum approach has been developed from an existing finite element code by Byrne and Duncan. The discontinuous rock body is modelled using solid elements, the properties of each element defined in terms of some combination of the properties of the intact rock and those of the joints. Jointed rock has been modelled as two-dimensional 4 noded quadrilateral element with two degrees of freedom at each node. The elastic properties of these elements have been represented by the empirical relationships given by Eqs.1 to 8 for both the intact rock and jointed rock

These empirical relationships have been incorporated into the finite element code. Theoretically, Poisson's ratio (ν) may range between 1 and 0.5. The practical range for soils and rocks is 0 to 0.5, a maximum value of 0.495 is used in the analysis.

Non-linearity in the finite element analysis has been incorporated using incremental method. The load is increased in series of steps, or increments. At the beginning of each new increment of loading an appropriate modulus value is selected for each element on the basis of the values of stress in that element using Eqs.1 and 7. The non-linear stress-strain behaviour is approximated by a series of straight lines. The principal advantages of this procedure are its complete generality and its ability to provide relatively complete description of the load deformation behaviour. Initial stress may be readily accounted for, as tangent modulus is expressed in terms of the stresses only. The principal limitation of this method is that it is not possible to simulate a stress-strain relationship in which stress decreases beyond peak. To simulate strain softening behaviour one would require to use a negative value of the modulus, and this is not possible in this method of analysis. For each load increment the modulus value of the jointed rock has been calculated using Eq.7.

5.0 ANALYSIS AND RESULTS

The finite element analysis has been carried out for different confining pressures (σ_3). The confining pressures have been varied from 0 to 10 MPa. The lower boundary of the jointed rock is fixed and the axial load is applied on the upper boundary. The load is applied in increments till the rock fails. Each increment is analysed twice, first time using the moduli values for the elements based on the stresses at the beginning of the increment, and the second time using moduli values based on the average stresses during the increment. The changes in stress and strain of the elements and the changes in the nodal displacements during each increment are added to the values at the beginning of the increment. During the loading if the element is found to fail in shear, the same is noted, but no changes are effected, and the element is allowed to follow the hyperbolic relation as before, in keeping with the non-linear elastic formulation of the problem. For the elements which fail in tension, the same is assigned very small values of Elastic modulus for the subsequent loads.

The finite element analysis can be carried out for any type of jointed medium. For the sake of comparison with the experimental data, finite element analysis has been carried out for three type of rock materials namely Plaster of Paris, Sandstone, Granite for both intact and jointed specimens. The material properties of these three rock materials are as given in Table 1. The stress and strain for each load increment has been plotted for different values of confining pressure (σ_3) for both intact and jointed specimens with different orientation of joints. The results have been compared with the experimental results of Yaji (1984) as given in Figs. 3 to 8.

Table 1: Material Properties of the Rock Materials

Property	Plaster of Paris	Sandstone	Granite
Mass density (KN/m^3)	12.25	22.5	26.5
Uni-axial compressive strength (MN/m^2)	9.5	70	123
Modulus number (k)	7200	45000	107000
Modulus exponent (n)	0.343	0.115	0.044
Cohesion (MN/m^2)	2.17	14.0	25.5
Angle of Internal Friction (degrees)	40.5	44.0	46.5
Classifcation	Soft Rock	Hard Rock	Extremely hard rock

The results compare well within the limit of theoretical and experimental framework for Plaster of Paris, Sandstone and Granite. Some typical results showing the comparison between the experimental and finite element results are given in Figs. 3 to 8 for Sandstone and Granite for different confining pressure

(σ_3) and joint inclination angles. The equivalent continuum analysis nevertheless works for all kinds of jointed rock materials. The results obtained using the analysis for both intact and jointed medium for different confining pressures are in agreement with the experimental results. The results compare well for different joint inclination angles. The equivalent continuum analysis has been thoroughly tested for multiple joints with different joint orientations and for all kinds of rock masses.

6.0 CONCLUSIONS

It can be concluded that the jointed rock can be represented as an equivalent continuum having the material properties given by Eqs. 1 to 8. The accuracy of the results depend on how well the joint factor is defined for a particular jointed medium. The major advantage of this kind of analysis is for modelling real discontinuous systems provided the properties of the intact rock are known and the joint factor is estimated accurately so that it represents the real fabric of the multiple joints present in the system.

This analysis can also be extended for rocks having joints filled with gouge material. Though the exponential relationships for average limit are used in the analysis, the expressions for the lower and upper bound are also incorporated in the analysis. Depending on the type of problem and the factor of safety required, any one of the expressions can be chosen and the analysis can be performed.

For a field problem where lower factor of safety is required, the lower bound limit equations can be used. If the level of confidence for estimating the factor of safety falls between the upper and lower limits, then the average limit equations can be used.

The analysis can also be extended to axisymmetric problems and the field problems with minimum efffort.

7.0 REFERENCES

Arora, V.K. (1987) "Strength and Deformation Behaviour of Jointed Rocks", Ph.D. Thesis, IIT Delhi, India.

Barton, N. and Choubey, V. (1977). "The Shear Strength of Rock Joints in Theory and Practice", Rock Mechanics, Vol.10, No.1-2, pp 1-54.

Bieniawski, Z.T. (1968) "The Effect of Specimen Size on the Compressive Strength of Coal", Int. J.Rock.Mech.Min.Sci, 5, pp 325-335.

Brown, E.T.(1970)"Strength of Models of Rock With Intermittent Joints", Soil Mech.Fdns.Div., ASCE96(SM6), pp 1935-1949.

Brown, E.T. and Trollope, D.H. (1970) "Strength of Model of Jointed Rock", Soil Mech. Fdns. Div., ASCE96(SM2), pp 685-704.

Einstein, H.H. and Hirschfeld, R.C. (1973) " Model Studies in Mechanics of Jointed Rocks", J.Soil Mech. Fdns. Div., ASCE99, pp 229-248.

Gerrard C.M.(1982). "Joint Compliance's as a Basis for Rock Mass Properties and the Design of Supports", Int. J.Rock. Mech. Min. Sci. and Geomech. Abstr. 19, pp 285-305.

Hoek, E. and Brown, E.T. (1980) "Underground Excavations In Rock", Institutioin of Mining and Metallurgy, London, pp 527.

Holland, K.L., and Lorig, L.J. (1997) "Numerical Examination of Empirical Rock-Mass Classification Systems", Int. J.Rock. Mech. Min. Sci. and Geomech. Abstr. Vol.34, NO.3-4, Paper NO.127.

Janbu, N. (1963) "Soil Compressiblity as Determined by Odometer and Triaxial tests", European Conf. Soil. Mech. Foundn. Engg., Wiesbaden, Vol.1, pp 19-25.

Kulatilake, P.H.S.W., He, W., Um, J., Wang, H. (1997) "A Physical Model of Jointed Rock Mass Strength Under Uniaxial Compressive Loading", Int. J. Rock. Mech. Min. Sci. and Geomech. Abstr. Vol.34, No.3-4, Paper NO.165.

Ramamurthy, T. (1994) "Strength and Modulus Responses of Anisotropic Rocks", Rock Engineering by Hudson J.A., Vol.1, Chapter 13, pp 313-329.

Roy, N. Ramamurthy, T Kate, J.M. (1995) "An Experimental Study of Engineering Behaviour of Jointed Mass in Uniaxial Compression", IGC, Vol.1, pp 69-72.

Yaji, R.K. (1984) "Shear Strength and Deformation of Jointed Rocks", Ph.D. Thesis, Indian Institute of Technology, Delhi, India.

Zienkiewicz, O.C., and Pande, G.N. (1977). "Time Dependent Multilaminate Model of Rocks - Numeical Study of Deformation and Failure of Rock Masses", Int. J. Numerical Anal. Methods. Geomech, 1, pp 219-247.

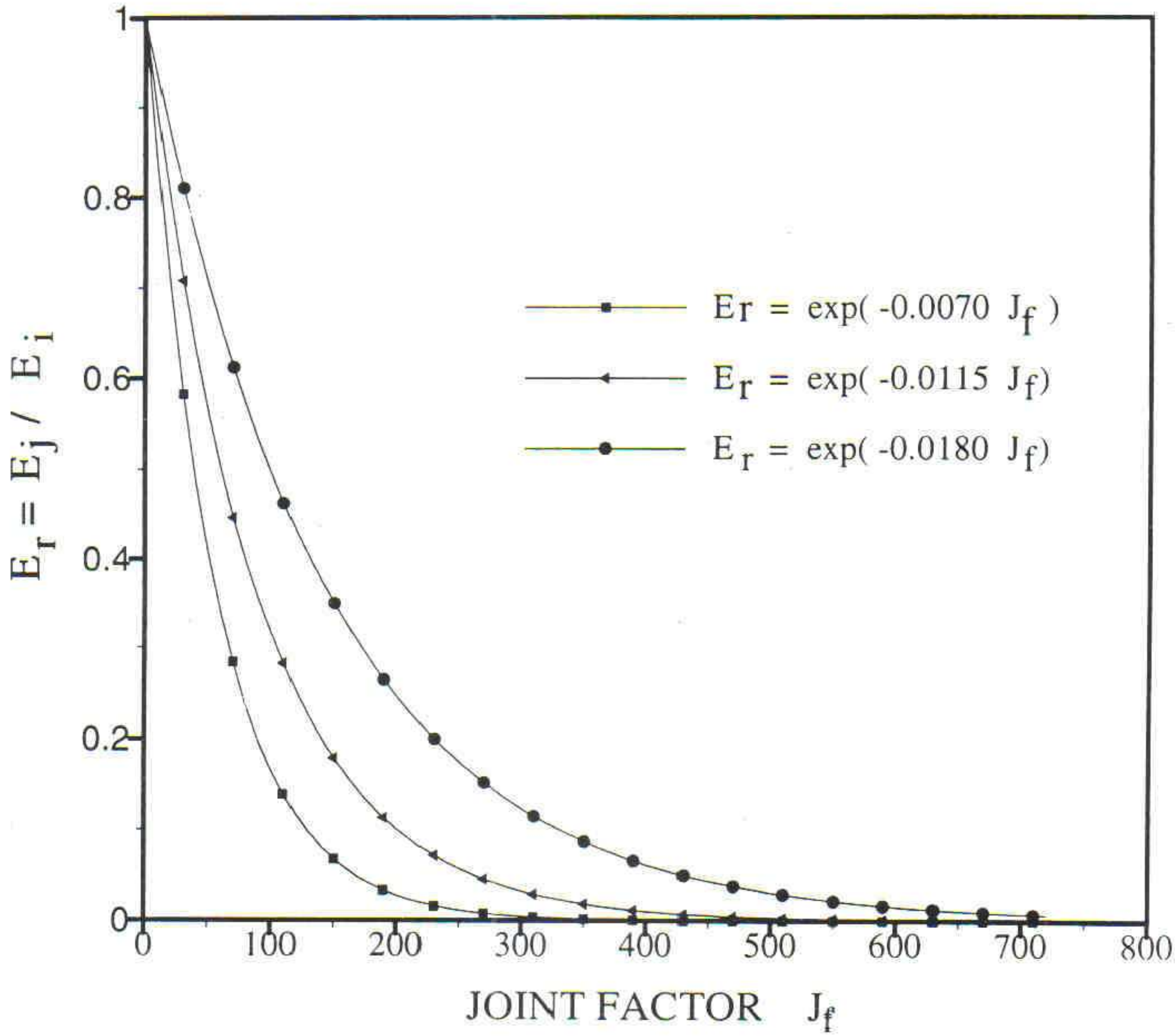

FIG 1 : EFFECT OF JOINT FACTOR ON RATIO E_r

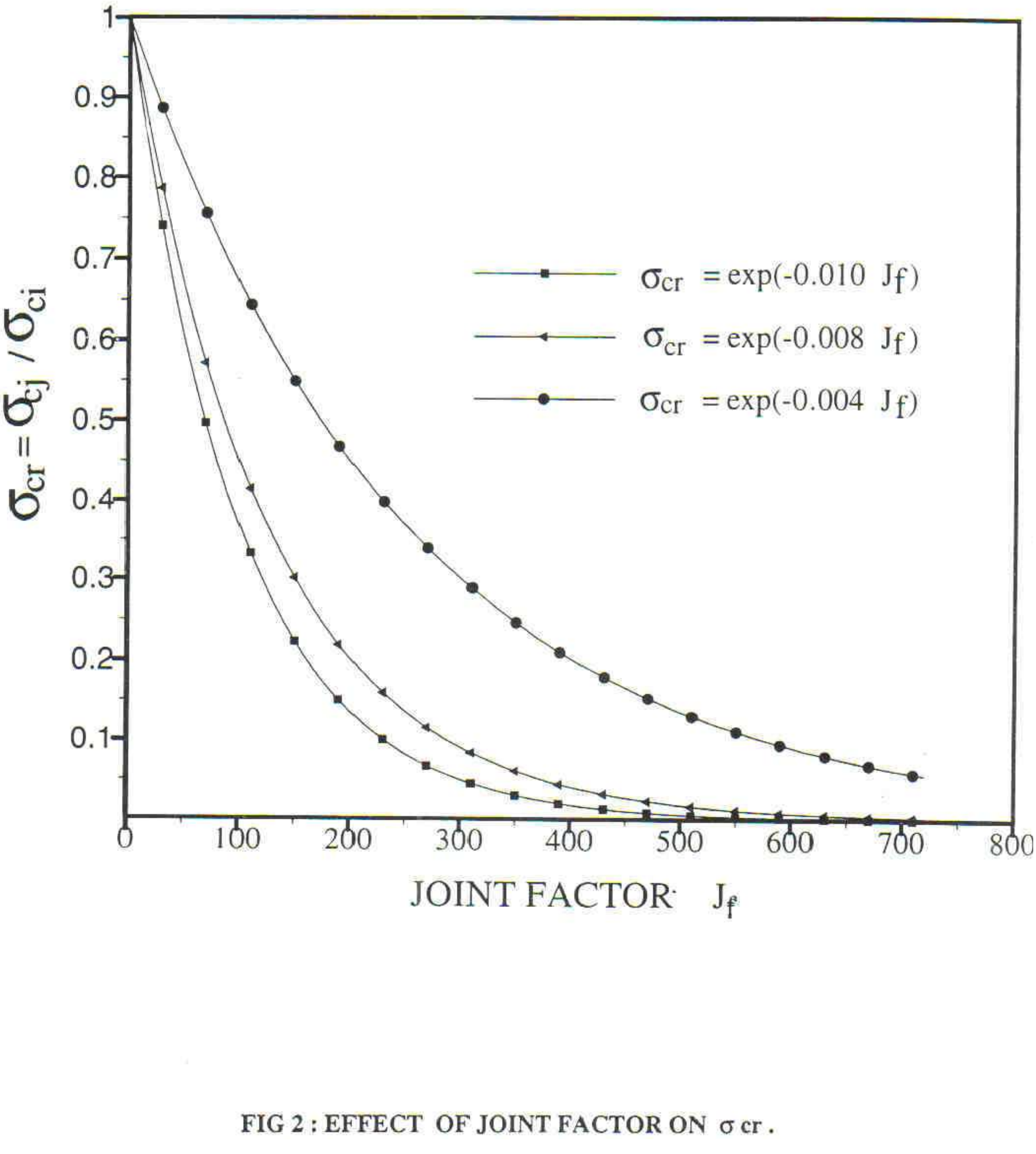

FIG 2 : EFFECT OF JOINT FACTOR ON σ cr .

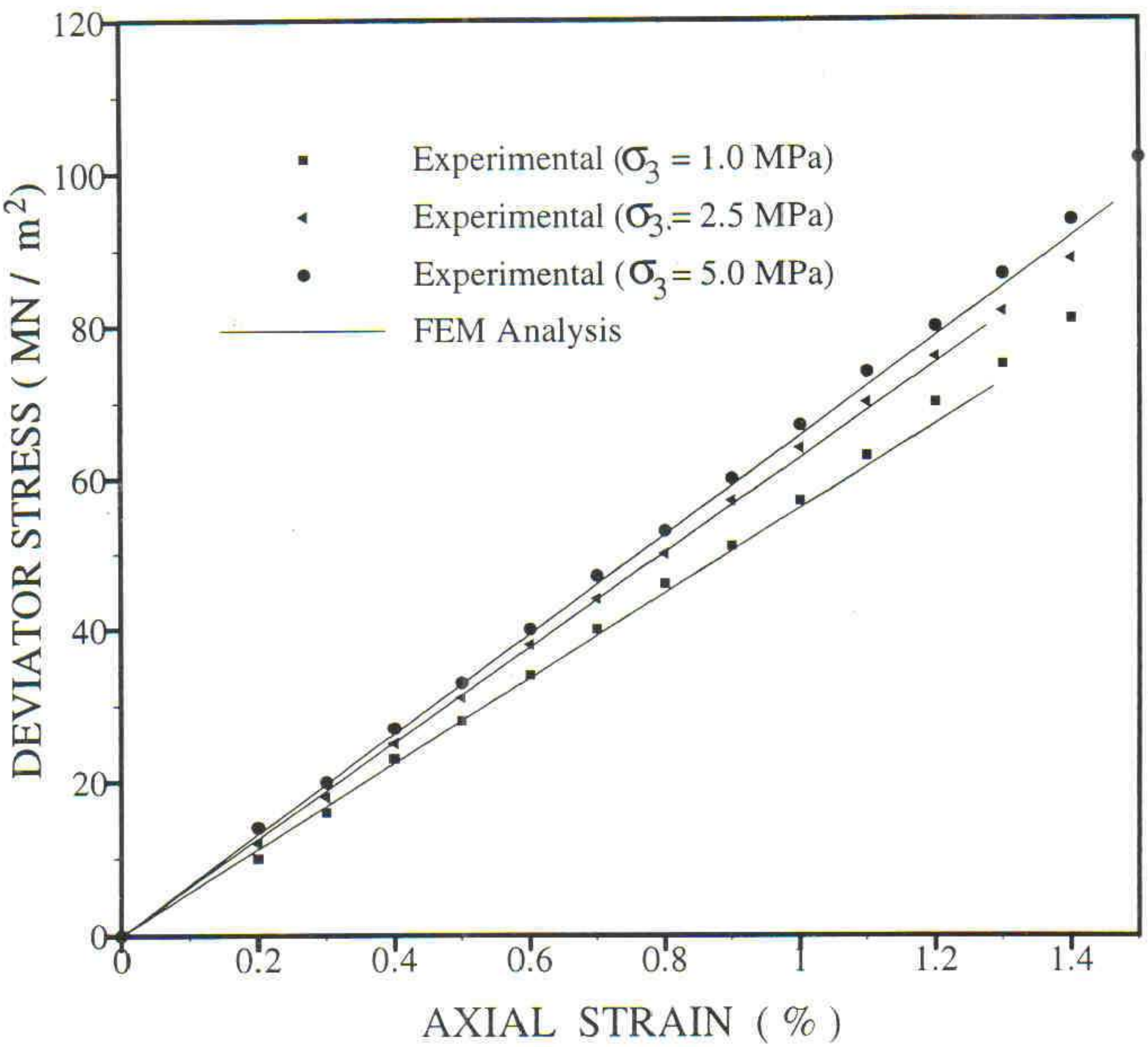

FIG 3: INTACT SPECIMEN OF SANDSTONE

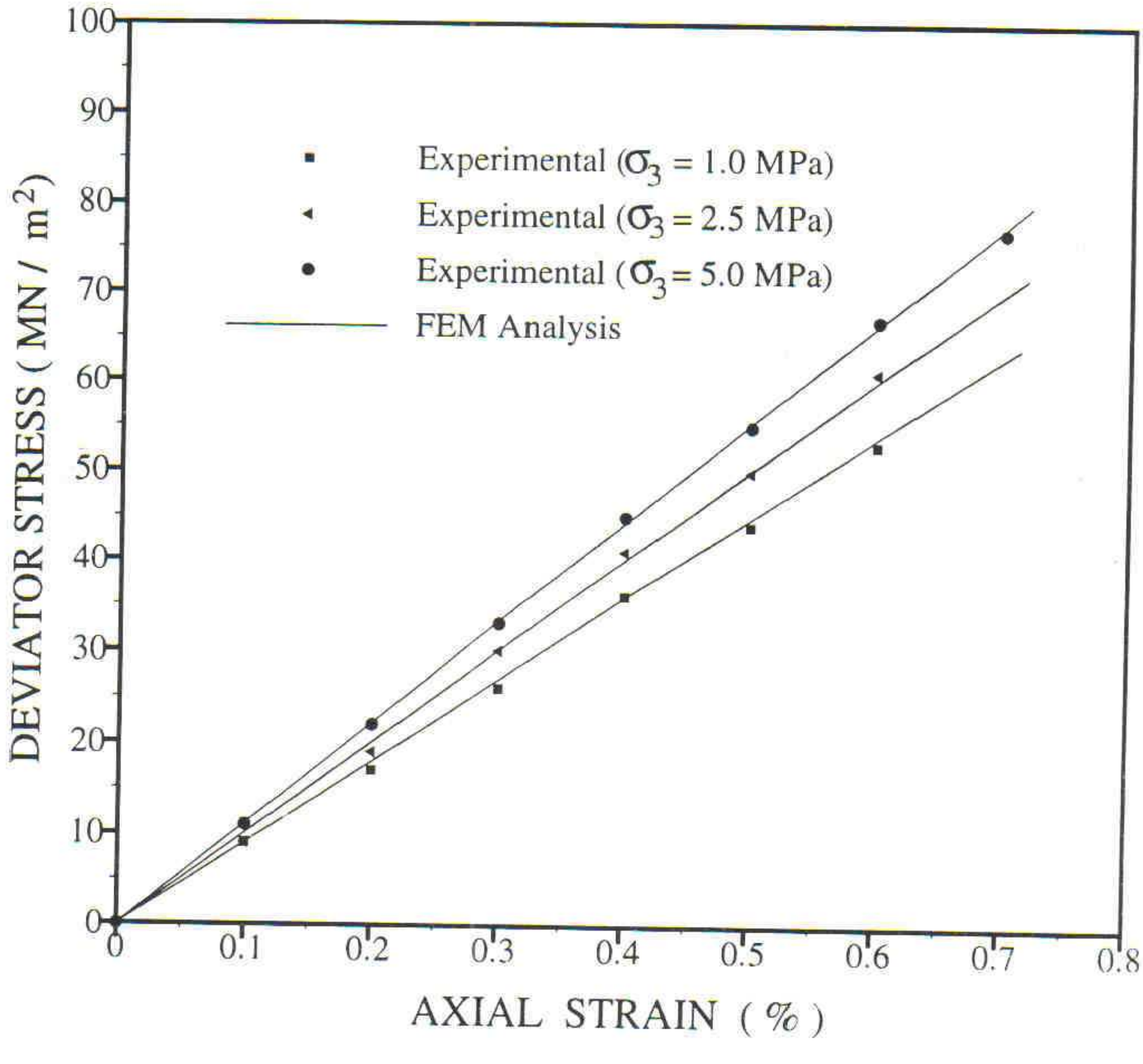

FIG 4 : SANDSTONE WITH JOINT INCLINATION ANGLE OF 0°

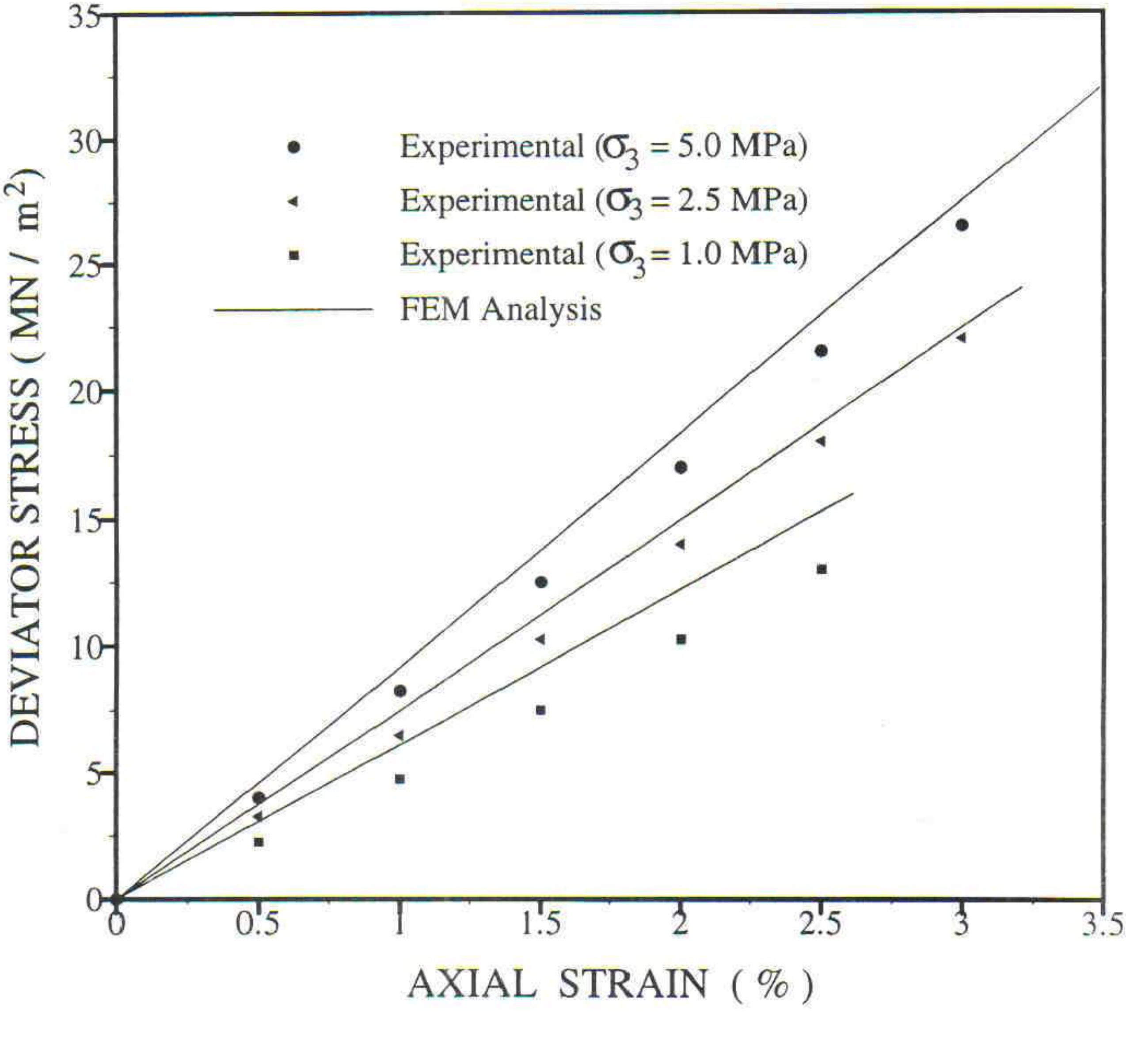

FIG 5 : SANDSTONE WITH JOINT INCLINATION ANGLE OF 45°

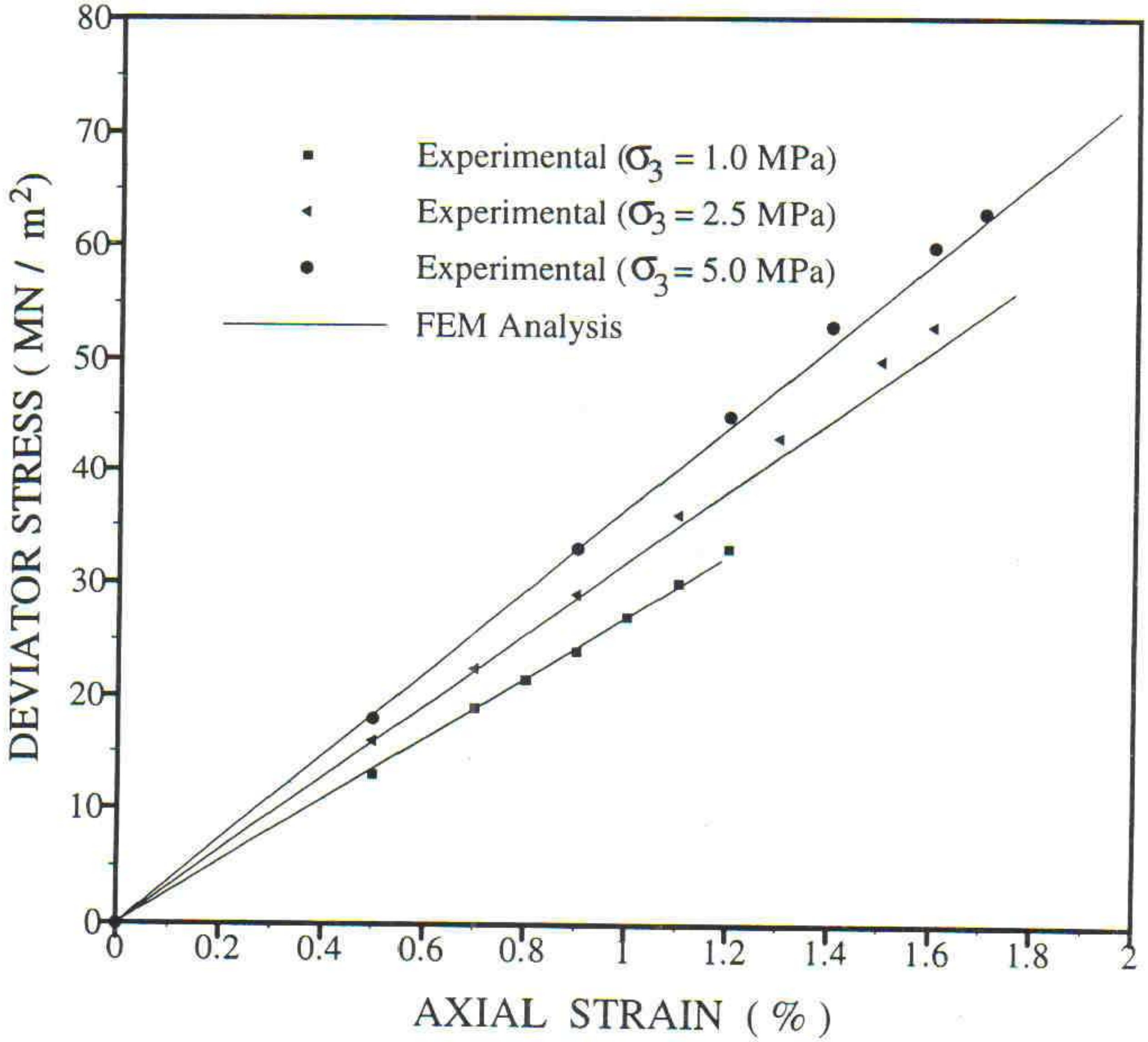

FIG 6 : SANDSTONE WITH JOINT INCLINATION ANGLE OF 60°

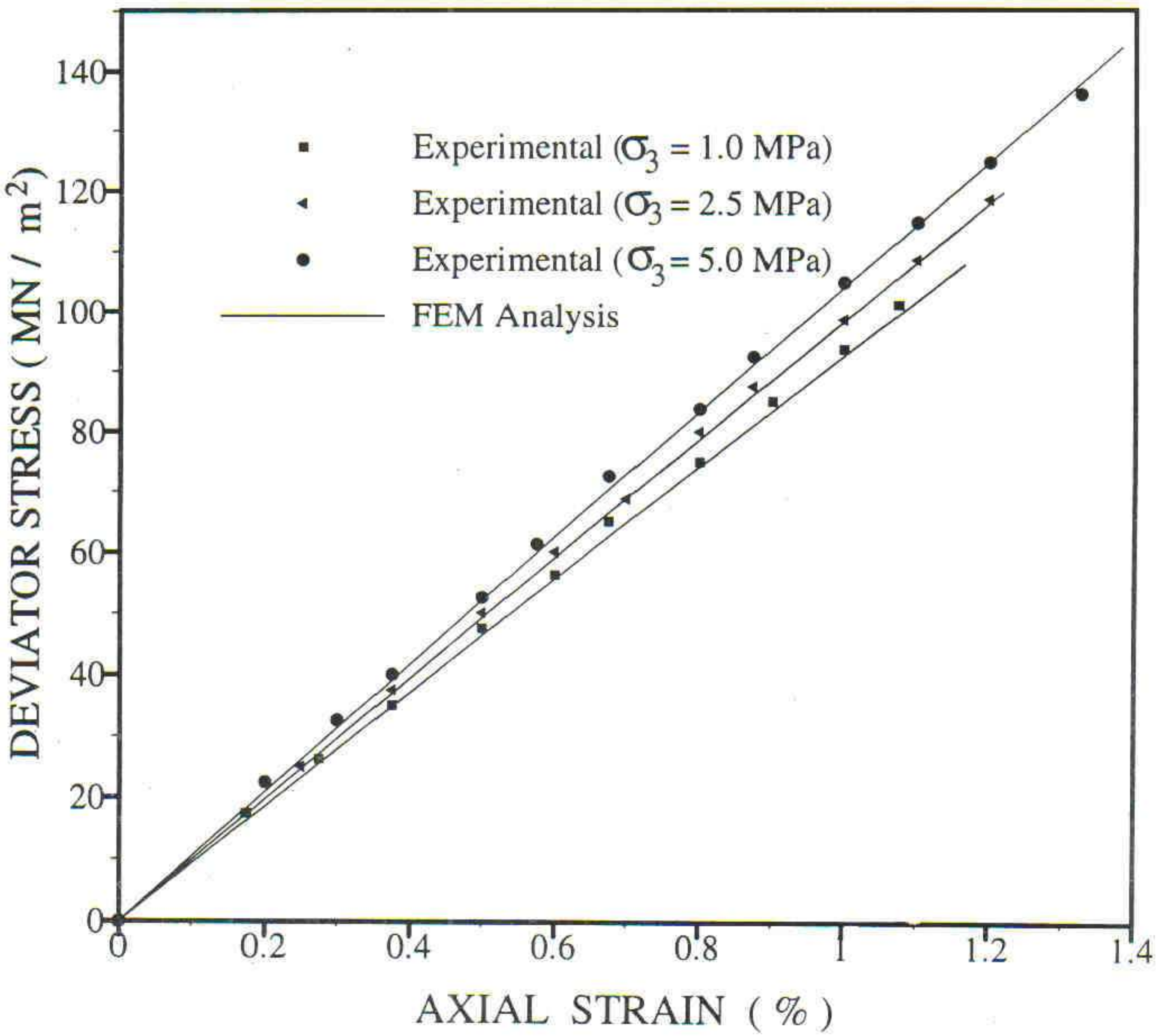

FIG 7 : GRANITE WITH JOINT INCLINATION ANGLE OF 75°

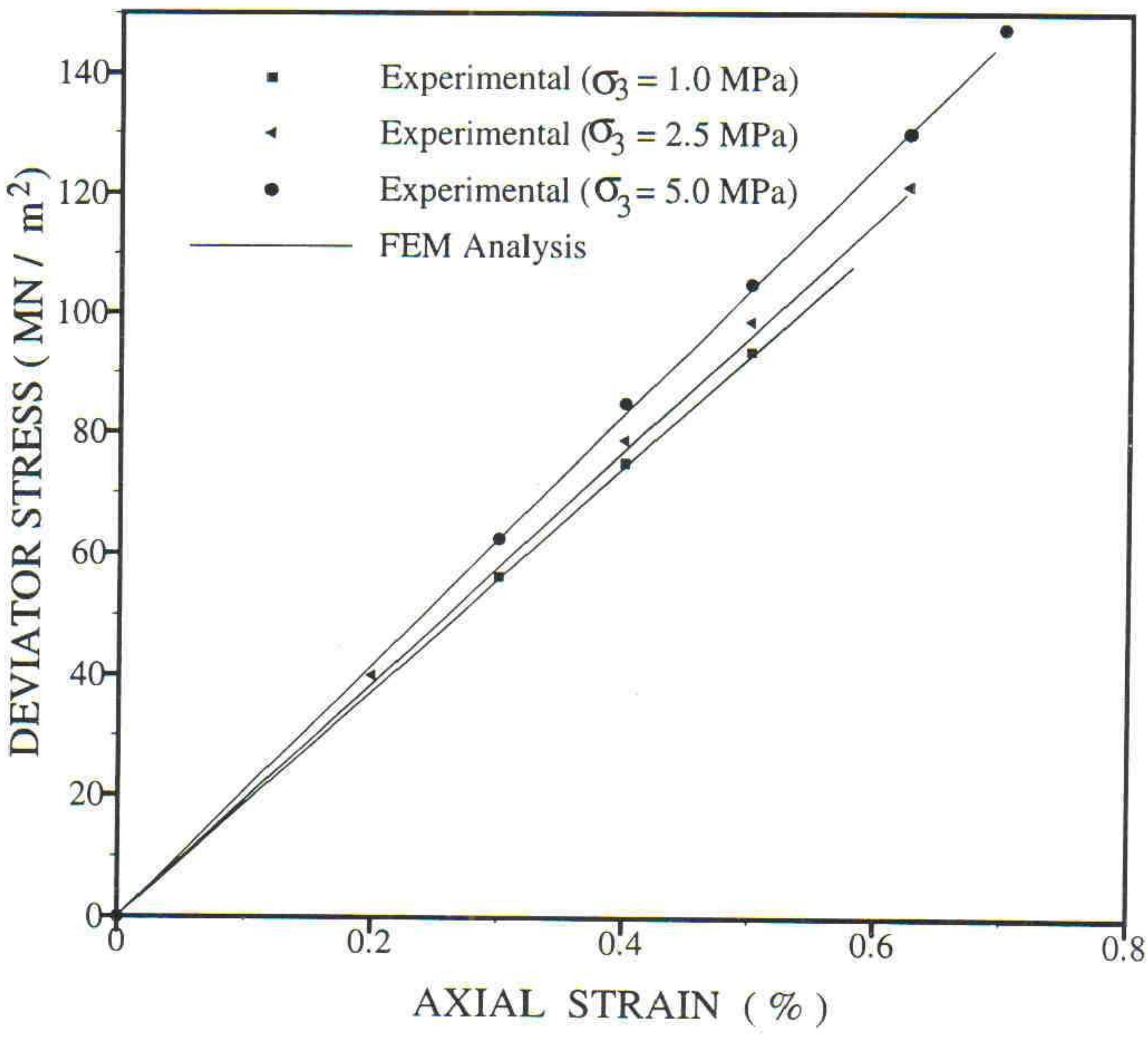

FIG 8 : GRANITE WITH JOINT INCLINATION ANGLE OF 90^{O}