Analysis of Rock Bolts in Bedded Strata

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ABSTRACT

This paper deals with the analysis of rock bolts in bedded strata involving the deep beam concept. It is assumed that there exists a firm competent strata above the rock beds such that the rock bolts may be anchored into this firm strata. Thus the beds which would have otherwise deflected individually and caused separation between beds ultimately resulting in roof falls, are now held together against the firm strata above. This composite member would now behave as a deep beam. As a result, the bending stresses in the beam are reduced and stability of the opening is assured. The deflection in the beam subjects the bolts to tension. It is assumed that no slip takes place between the beds and the transfer of stresses into the bolt due to deflection of the beam is perfect. Since the beam is suspended to the firm strata above it, the transverse loading on it is the self weight of the beam alone and it is assumed that the horizontal insitu stresses along the span direction of the beam are small and do not play a role in the deformation of the beam.

The differential equation for the Timoshenko beam is modified to incorporate the bolt forces. The equations are solved exactly using Laplace Transforms to obtain the bolt forces. A parametric study brings out the effect of the span of the opening, thickness of the beam, number and diameter of bolts on the bolt force, and deflection and bending stresses in the beam. Charts are provided which can be used for the design of openings using rock bolts.

Key words : *Bedded strata, Timoshenko beam, Rock bolts, Laplace Transforms, Suspension effect, Bolt forces, Bending stresses, Deflections*

1.0 INTRODUCTION

Rock bolts are being extensively used in underground excavations primarily to increase the stiffness and strength of the rockmass. Bolting is used passively to connect together several components of the rockmass. They are used to connect layered strata above roofs in order to form deep beams.

Situations arise wherein tunnels are required to be driven through bedded strata where the layers are very thin. Beyond a few thin layers there might exist a competent strata. As tunneling operations are in progress, these thin beds would deflect individually, separate from one another and cause damage to the opening owing to their low moment of resistance. Under such circumstances, these layers may be connected together passively by bolting them and the bolt anchored securely into the competent strata. The feature of this mechanism is the creation of a deeper beam with a higher moment of resistance.

One of the methods of analysis applied to composite beams is the method of transformed sections [Timoshenko, (1976)]. Analysis of rock bolt systems have been attempted by a number of researchers [Newmark, el.al. 1951; Oreste and Peila; 1996 and Stimpson, 1983]. In this paper the Timoshenko beam theory is used to analyze the deep beam created by bolting the bedded strata into the competent strata above. It is assumed that all the bolted beds are of the same rock type and there exists perfect bonding between the bolt and rock.

2.0 PROBLEM DEFINITION AND FORMULATION

The problem consists of an opening of span 'l', the roof of which consists of layers of sedimentary beds up to a height 'h', beyond which exists a competent strata. These beds are connected together using rock bolts of diameter 'd' which would extend into the competent strata (Fig.1). Rock bolts are installed symmetrically over the span of the opening. The spacing of the bolts in the plane of the opening is 'b'.

By bolting the beds and anchoring the bolt into the competent strata we have created a deep beam. The behaviour of this beam is similar to a continuous beam except that the props created by the bolts are not rigid but are subjected to deflections equivalent to the elongation of the bolts. The tensile forces developed in the bolts will depend upon the elongation of the bolts. Since the beam is thick, it will now experience shear deformations and hence the Timoshenko beam equation is used to analyse the beam. The tensile forces developed in the bolt are included in the governing differential equations.

The governing differential equation for the Timoshenko beam with a single bolt force is as follows

$$GAk\left[\frac{d^2w}{dx^2}-\left(\frac{d\phi}{dx}\right)\right]=-q+F_1\delta(x-x_1) \quad [1]$$

$$EI\frac{d^2\phi}{dx^2}+GAk\left[\frac{dw}{dx}-\phi\right]=0 \quad [2]$$

where, F_1 is the bolt force acting at a distance x_1 from origin.

q is a uniformly distributed load due to the self weight of the beam

w is the deflection of the beam

ϕ is the slope

δ is a delta function which implies $\delta = 0$ when $x \le x_1$

$\delta = 1$ when $x > x_1$

G is the shear modulus of the rock beds, A is the area of cross section of the beam and k is factor to accommodate for the non-uniform shear stress distribution at a section in a one-dimensional approach which is a function of the Poisson's ratio of the rock. For a rectangular cross section $k = 10(1+\nu)/(12+11\nu)$

EI is the flexural rigidity of the beam. G may be expressed in terms of the elastic modulus and Poisson's ratio as $G = E/(2(1+\nu))$

Equations 1 and 2 are solved using the Laplace Transformations. Four boundary conditions are required to solve the equations as a function of the bolt force and one to evaluate the bolt force. It may be assumed that the beam is simply supported. By making this assumption we would be erring on the conservative side only because due to blasting, the immediate rock may be in the disturbed state and hence its strength reduced at the support of the beam, i.e. at the haunches of the opening.

The boundary conditions are

Moment $\frac{d\phi}{dx} = 0$ at $x = 0$ and $x = l$.

Deflection $w = 0$ at $x = 0$ and $x = l$.

The solution for the deflection and slope of the beam are

$$w=-\frac{EI}{GAk}\left[c_3x+\frac{qx^2}{2EI}-\frac{F_1(x-x_1)}{EI}U(x-x_1)\right]+c_1x+\frac{c_3x^3}{6}+\frac{qx^4}{24EI}-\frac{F_1(x-x_1)^3}{6EI}U(x-x_1) \quad [3]$$

$$\phi = c_1 + \frac{c_3 x^2}{2} + \frac{qx^3}{6EI} - \frac{F_1 (x - x_1)^2}{2EI} U(x - x_1) \quad [4]$$

where, $U(x\text{-}x_1) = 0$ for $x \le x_1$

$U(x\text{-}x_1) = 1$ for $x > x_1$

$$c_1 = \frac{ql^3}{24EI} - \frac{F_1}{6EIl}\left[l^2 (l - x_1) - (l - x_1)^3\right]$$

$$c_3 = \frac{-ql}{2EI} + \frac{F_1 (l - x_1)}{EIl}$$

Equations. 3 and 4 have been derived for a single bolt at x_1 from the origin. The general equation for deflection for n bolts is

$$w = -\frac{EI}{GAk}\left[c_3 x + \frac{qx^2}{2EI} - \sum_{i=1}^{n} \frac{F_i (x - x_i) U(x - x_i)}{EI}\right] + c_1 x + c_3 \frac{x^3}{6} + \frac{qx^4}{24EI} - \sum_{i=1}^{n} \frac{F_i (x - x_i)^3 U(x - x_i)}{6EI} \quad [5]$$

where,

$$c_1 = \frac{ql^3}{24EI} - \frac{1}{6EIl} \sum_{i=1}^{n} \{F_i l^2 (l - x_i) - F_i (l - x_i)^3\}$$

$$c_3 = \frac{-ql}{2EI} + \sum_{i=1}^{n} F_i \frac{(l - x_i)}{EIl}$$

x_i corresponds to the location where the bolt force F_i acts.

Equation 5 gives the expression for the deflection of the beam due to the self weight of the beam and the bolt forces. In order to solve for the bolt forces one needs to satisfy the displacement compatibility at the location of the bolts. Hence the deflection at the points where the bolts are installed are equated to the elongations of the bolts at those locations. It is assumed that the variation of tension in the bolt is linear. Hence

$$w_i = \frac{F_i L_b}{2 A_b E_b} \quad [6]$$

where, L_b is the length, A_b is the area and E_b is the elastic modulus of the bolt. While considering the length of the bolt, it is assumed that 10% of the length is taken into the competent strata. Hence

$L_b = 1.1h$ where h is defined in Fig. 1.

Substituting for the elongations at all the n locations one obtains n equations in n unknown bolt forces which can be solved simultaneously by the Gauss Elimination technique. Once the bolt forces are known, the deflection along the span of the beam can be evaluated from Equation 5. The moment along the span of the beam may be evaluated from

$$M = -EI\, d\phi/dx \qquad [7]$$

The uniform load q on the beam due to its self weight may be expressed as

$$q = \gamma\, b\, h \qquad [8]$$

where γ is the unit weight of rock.

3.0 RESULTS AND DISCUSSION

A parametric analysis is carried out to study the effect of the span of the opening, thickness of the bedded strata, diameter of bolt and the number of bolts on the maximum bolt force, maximum bending stress and deflection of the beam. All parameters considered and evaluated are in dimensionless form. The range of parameters considered are as follows

Dimensionless diameter of bolt,	d/l	=	0.0025, 0.0032, 0.005 and 0.0064
Dimensionless thickness of strata,	h/l	=	0.01 to 1.0
Modular ratio,	E_b/E	=	7, 21,70 and 210
Number of bolts	n	=	1, 2 and 3
Spacing of bolts in plane of opening	b/l	=	0.025, 0.05, 0.1 and 0.2

The ratios of d/l considered would correspond to a bolt diameter of 25mm and 32mm for opening spans of 5m and 10m. The ratios of E_b/E correspond to a E for the rockmass in the range of 30 GPa and 1 GPa where E of steel is 210 GPa.

The deflected shape of the beam is presented first as it helps in explaining the reasons for the variations in bolt forces and the bending stresses in the beam presented later. Figs. 2, 3 and 4 show the deflected shape of the beam for different h/l values and number of bolts varying from 1 to 3. The deflection of the beam is expressed in dimensionless form as $wE/\gamma l^2$. The constant parameters are $E_b/E = 21$, b/l=0.2 and d/l=0.005. The figures show that for low values of h/l the deflection of the beam is non uniform and a kink is observed at the point where the bolts are installed. As the h/l increases, the deflection tends to be more uniform. Until these deflections acquire a uniform profile the maximum deflection is away from the center for all values of n. The point of maximum deflection approaches the center of the beam with increase in h/l values until a uniform deflected shape of the beam is achieved. Beyond this particular value, of h/l, the beam deflects by a lesser amount, the maximum deflection occurring at the center of the beam. The deflections decrease with increase in n.

A similar plot for $E_b/E = 210$ and n=1 and all other parameters remaining the same is shown in Figure 5. Here the beam undergoes lesser deformations as compared to a stiffer beam. Nevertheless, the pattern of the deflected shape remains the same as in Figure 4.

The variation of the maximum deflection, $w_{max}E/\gamma l^2$ versus h/l is plotted in Fig. 6 for n varying from 1 to 3 and for E_b/E ratios of 21 and 210. The ratios b/l and d/l are kept constant at 0.2 and 0.005 respectively. It is seen that at very low values of h/l the deflections are very high. These deflections drop down rapidly and again reach a maximum at a h/l of around 0.2 and 0.35 for E_b/E =21 and 210 respectively, beyond which the maximum deflections decrease monotonically. This behaviour can be explained from Figs. 2 to 5. For very low values of h/l the beam deflects non uniformly and the maximum deflection is high and away from the centre of the beam. As h/l increases deflections become uniform and there is an increase in the maximum deflection up to a certain value of h/l and beyond this value the overall deflection of the beam itself decreases. Fig. 6 depicts that the maximum deflection decreases with increase in n. With E_b/E increasing from 21 to 210 the maximum deflections are smaller and these deflections tend to be more uniform. From this figure it is clear that it is always economical to operate for h/l>0.1. The maximum of the maximum deflections in the beam is of the order of 2.62 and 1.89 for n=1 and 3 for $E_b/E = 21$ and occurs at a h/l of 0.18 and 0.22 respectively. The corresponding values for $E_b/E = 210$ are 0.9 and 0.7 respectively for n=1 and 3 and occur at a h/l of 0.36 and 0.45 respectively.

Figure 7 shows the variation of the dimensionless maximum bolt force $F/\gamma l^3$ with the thickness of the beam h/l for various E_b/E ratios for n=1, b/l=0.2 and d/l=0.005. It is seen that the maximum force increases up to h/l = 0.10 for E_b/E=7 and h/l=0.24 for E_b/E = 210 and then decreases rapidly with h/l. The maximum value of the normalised maximum bolt force corresponding to E_b/E of 210 is of the order of 0.0224. Considering $\gamma = 27 kN/m^3$ and a span of 5m this value would correspond to a bolt force of 75.6 kN which is quite small. This is because the loading on the beam is its self weight alone and there is no other loading on it.

Figure 8 shows the variation of $F/\gamma l^3$ with h/l for number of bolts varying from 1 to 3 and d/l = 0.005 and b/l = 0.2. Plots are given for E_b/E = 21 and 210. The maximum bolt force peaks up for h/l in the range of 0.13 to 0.16 for $E_b/E = 21$ and 0.24 to 0.3 for $E_b/E = 210$ and then drops rapidly. The reduction in the maximum bolt force is high when the number of bolts are increased from 1 to 2 while it is marginal for n increased to 3. Further the real advantage of increasing the number of bolts is achieved for h/l in the range of 0.1 to 0.25. Beyond a h/l of 0.25 there is no advantage in increasing the number of bolts. It is seen from the plot that the curve for n=3 crosses the curve for n=2 at a h/l of 0.21 for $E_b/E = 21$, and the change is very marginal. This may be because the maximum bolt force which is carried by both the bolts for n=2 is at a distance of l/3 and 2l/3 while that for n=3 is carried by the central bolt positioned at l/2. The maximum value of $F/\gamma l^3$ for n=1 is 0.0125 for $E_b/E = 21$ and 0.0224 for $E_b/E = 210$.

The variation of $F/\gamma l^3$ with h/l for various values of b/l and for d/l=0.005, $E_b/E=21$ and n=1 is depicted in Fig. 9. Here again it is seen that the maximum $F/\gamma l^3$ occurs for a h/l within a range of 0.08 and 0.3 Within this range there is a considerable reduction in $F/\gamma l^3$ with decrease in the out of plane spacing b/l of the bolts. This is true because as the spacing is reduced each bolt will have to carry a lesser load.

Figure 10 depicts the variation of $F/\gamma l^3$ with h/l for various diameters of the bolt, d/l for a b/l=0.2, $E_b/E=21$ and n=1. It is seen that a larger diameter bolt carries larger load. The maximum for all bolts occurs in the range of h/l = 0.1 to 0.16. Beyond this range the force drops rapidly.

Figure 11 depicts the variation of the maximum bending stress $\sigma_{max}/\gamma l$ with h/l for n=1, 2 and 3, $E_b/E = 21$ and 210, and for b/l = 0.2 and d/l = 0.005. The figure shows that the bending stresses are very high for very low values of h/l which is understandable because of its very low flexural rigidity. There is a threshold value of h/l beyond which the stresses again rise, reach a peak and drop down. This is because of the fact that the loading on the beam is only its self weight. The bending stresses are directly proportional to the self weight of the beam which is a function of h/l and inversely proportional to the second moment of area which is a function of the cube of h/l. As a result, one observes a minimum stress at h/l<0.2 and a maximum at h/l>0.2. It is also seen that for h/l<0.1 for $E_b/E=21$ and h/l<0.2 for $E_b/E=210$ the curves are not smooth. This is because the maximum bending stress does not occur at a constant position on the beam. The location varies and for h/l>0.1 for $E_b/E=21$ and h/l>0.2 for $E_b/E=210$ the maximum stresses occur at the midpoint of the beam. This shift is predominant for n=1 and n=3. For n=2 the maximum stresses occur at the midpoint of the beam even for low h/l values. This figure again proves the fact that it is always economical to operate at thicknesses where h/l>0.1. The maximum stresses are lesser for a higher E_b/E ratio.

These charts can be used to design excavations in thinly bedded strata and the number of bolts required to stabilize the opening. The effect of the horizontal insitu stresses are not considered in the analysis. The following example problem helps to explain the use of the charts.

4.0 EXAMPLE PROBLEM

Consider creating an excavation in bedded strata with the following parameters

Span of the opening = 5m
Thickness of the strata up to the firm layer = 1m
Elastic modulus of rock = 10 GPa
Unit weight of rock = 27 kN/m^3
Allowable tensile strength of rock = 0.22 MPa
Diameter of bolts = 25 mm
Spacing of bolts in the plane of excavation = 1 m

For these parameters the dimensionless parameters are h/l=0.2, b/l=0.2, d/l=0.005 and E_b/E=21.

Consider 1 bolt, i.e. n=1

From Figure 8 the value of $F/\gamma l^3$ =0.0096 which gives an F of 32.4 kN. The value of $\sigma_{max}/\gamma l$ = 1.95 from Figure 11 which gives a σ_{max} of 263 kPa which is greater than the allowable tensile strength of the rock. Figure 11 shows that for h/l=0.2 there is no improvement in the maximum bending stresses with n=2. Hence consider n=3.

For n=3, the value of $F/\gamma l^3$ = 0.007 which gives an F of 23.625 kN. The value of $\sigma_{max}/\gamma l$ is 1.48 which gives a σ_{max} of 200 kPa. This gives a factor of safety of 1.1. The maximum deflection $w_{max}E/\gamma l^2$ for n=3 is 1.86 which gives w_{max}= 0.125mm which is very small.

5.0 CONCLUSIONS

A methodology based on the deep beam theory has been proposed for the analysis of rock bolts in bedded strata. It is assumed that a firm strata lies above the thin layers of bedded strata into which the rock bolts would be anchored thus causing a suspension effect. It is also assumed that no slip takes place between the beds, and the bolts help in creating a monolithic deep beam. The effect of the horizontal insitu stresses are neglected in the analysis.

The Timoshenko beam equation is modified to incorporate the unknown bolt forces. The equations are solved exactly using Laplace transforms to obtain the bolt forces. Satisfying the displacement compatibility at the points where the bolts are inserted, the bolt forces are estimated. The effect of the size of opening, thickness of the strata up to the firm layer, number and diameter of bolts on the bolt forces, and deflection and bending stresses in the equivalent beam are studied. Based on the analysis the following conclusions are made.

(i) At very low values of h/l ($h/l<0.1$) the bending stresses are very high. The deflections are also of a very high order. Hence it is always advantageous to work with $h/l>0.1$.

(ii) At low h/l values ($0.1<h/l<0.4$ depending on the E_b/E ratio) the deflection in the beam is non-uniform and it becomes uniform with increase in h/l values. Thus there lies a h/l value where the deflection in the beam is maximum beyond which it drops gradually. This value of h/l depends on the E_b/E ratio, b/l and d/l.

(iii) Corresponding to these h/l ratios for the maximum of the maximum deflections the bolt forces reach a peak and drop down with increasing h/l ratios. The bolt forces are not very high because the loading on the beam is by its self weight alone.

(iv) The main advantage of this analysis lies in the computation of the bending stresses and maximum deflections in the beam. The effect of increasing the number of bolts is predominantly felt in the region of $h/l=0.1$ to 0.4 depending on the E_b/E ratio. Within this region, the bending stresses and deflections reduce with increase in the number of bolts. Beyond this region, the stresses and deflections do drop down but the effect of n is negligible.

(v) At larger values of h/l the bending stresses are small.

(vi) The charts provided help in deciding upon the number of bolts required to prevent failure of the roof.

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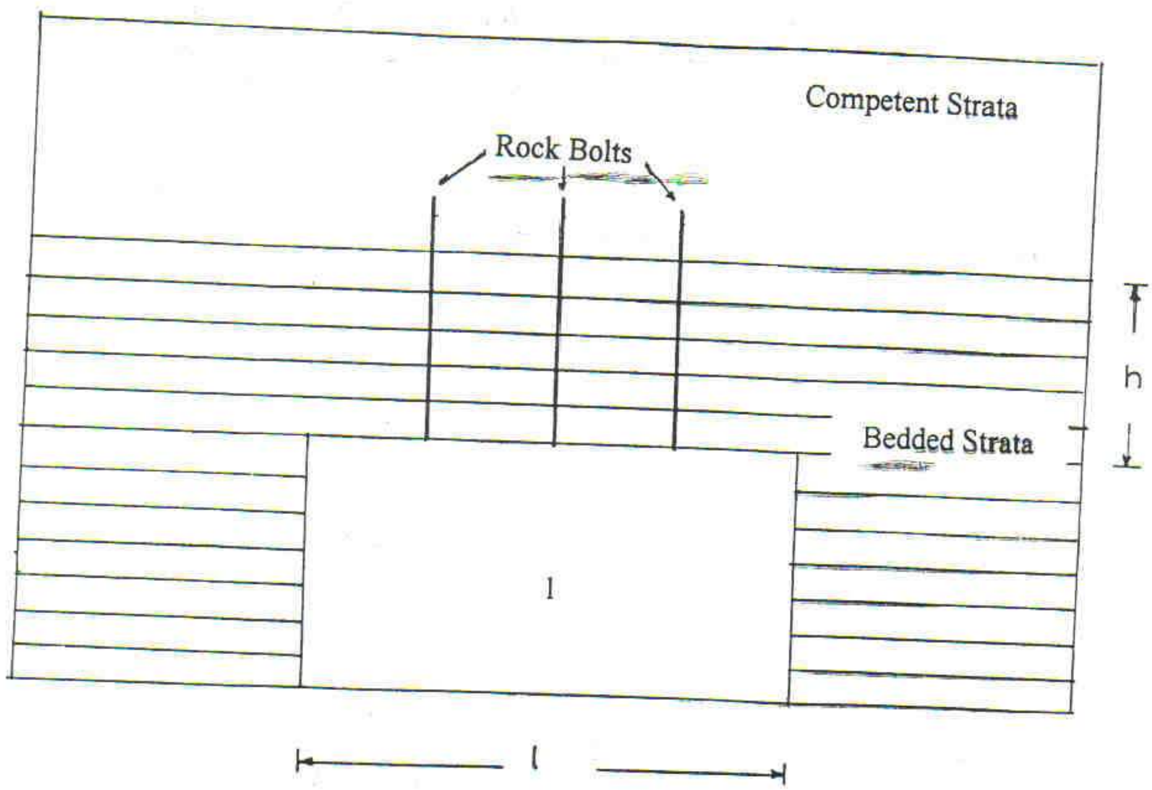

Fig. 1 Definition Sketch

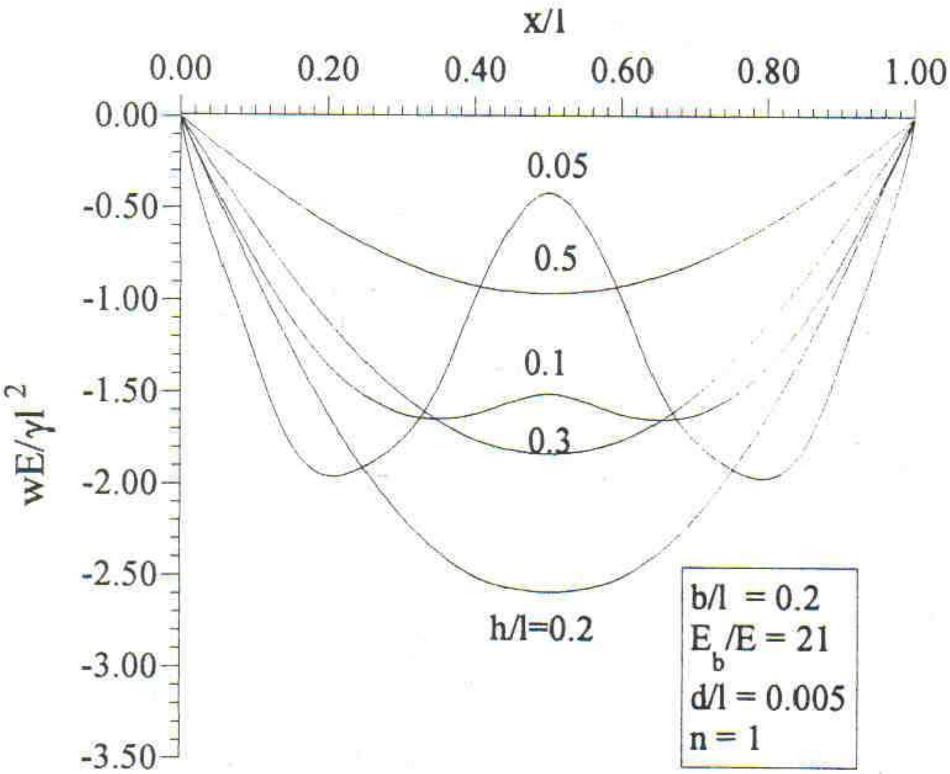

Fig. 2 Effect of h/l on Deflected Shape of Beam

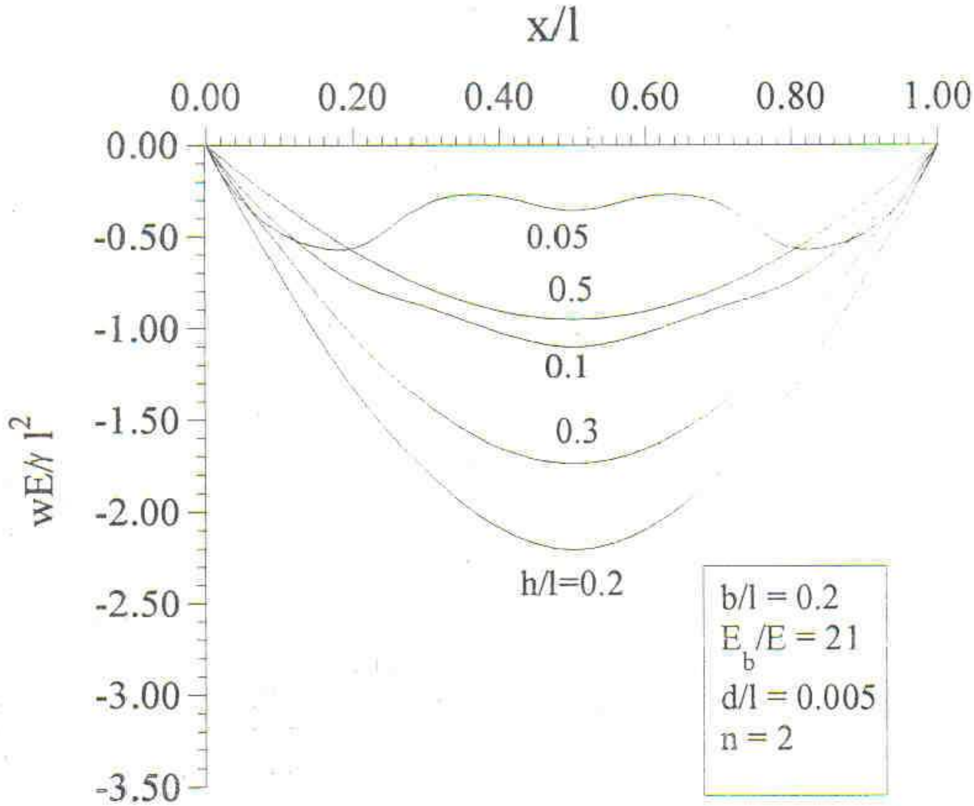

Fig. 3 Effect of h/l on Deflected Shape of Beam

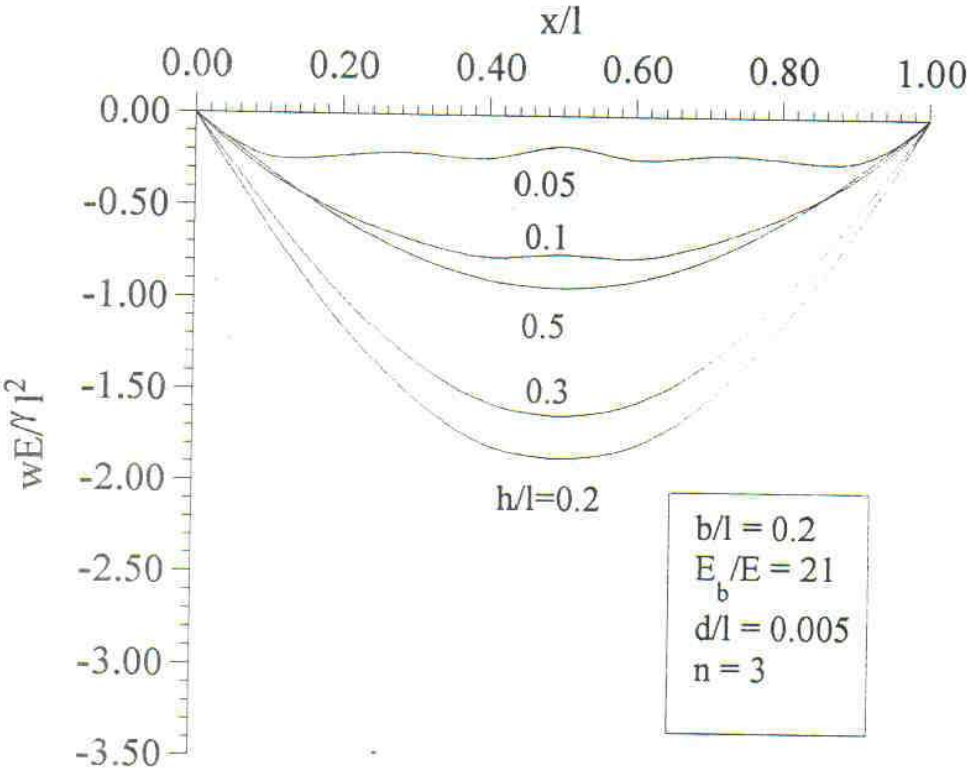

Fig. 4 Effect of h/l on Deflected Shape of Beam

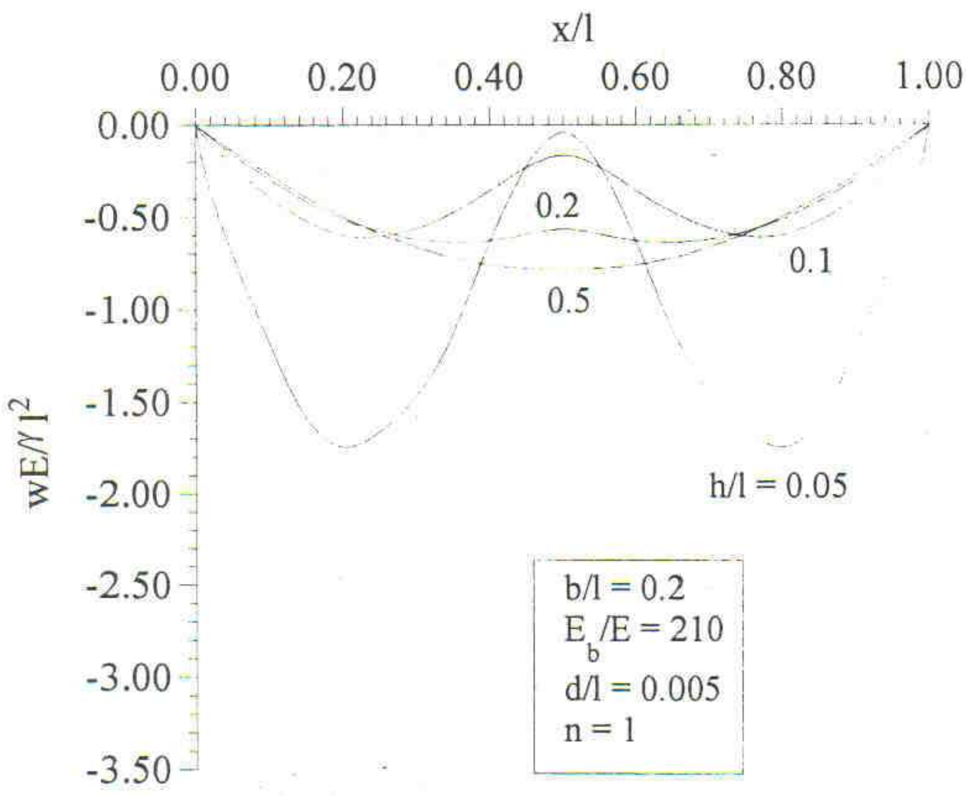

Fig. 5 Effect of h/l on Deflected Shape of Beam

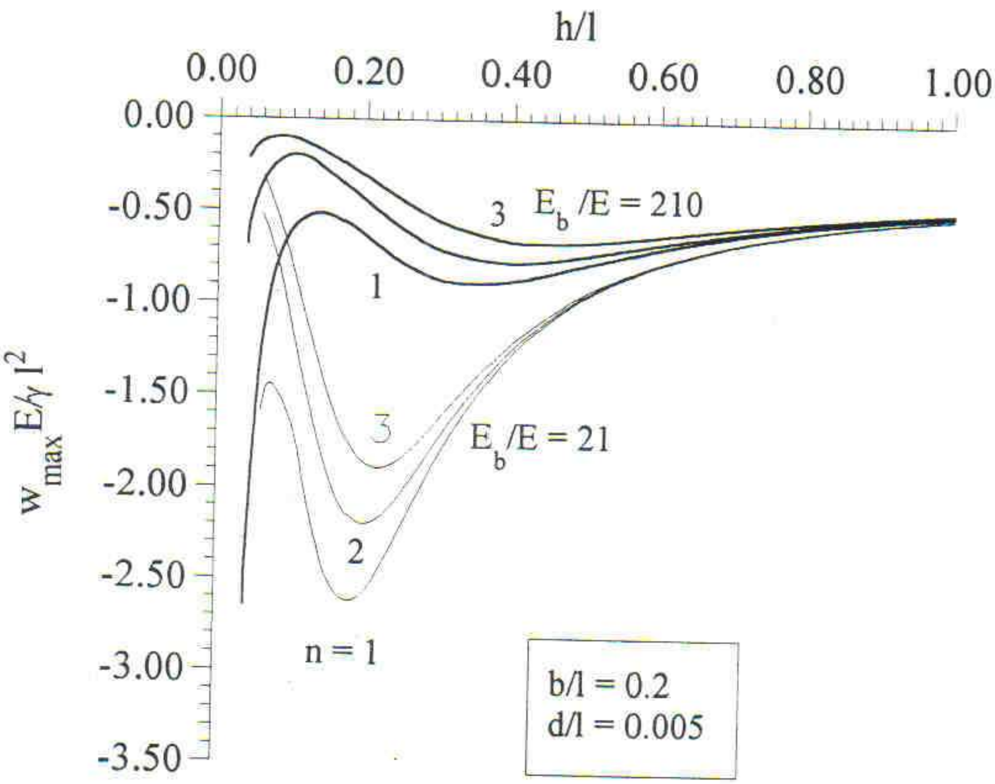

Fig. 6 Variation of Maximum Deflection with h/l for Different E_b/E and n

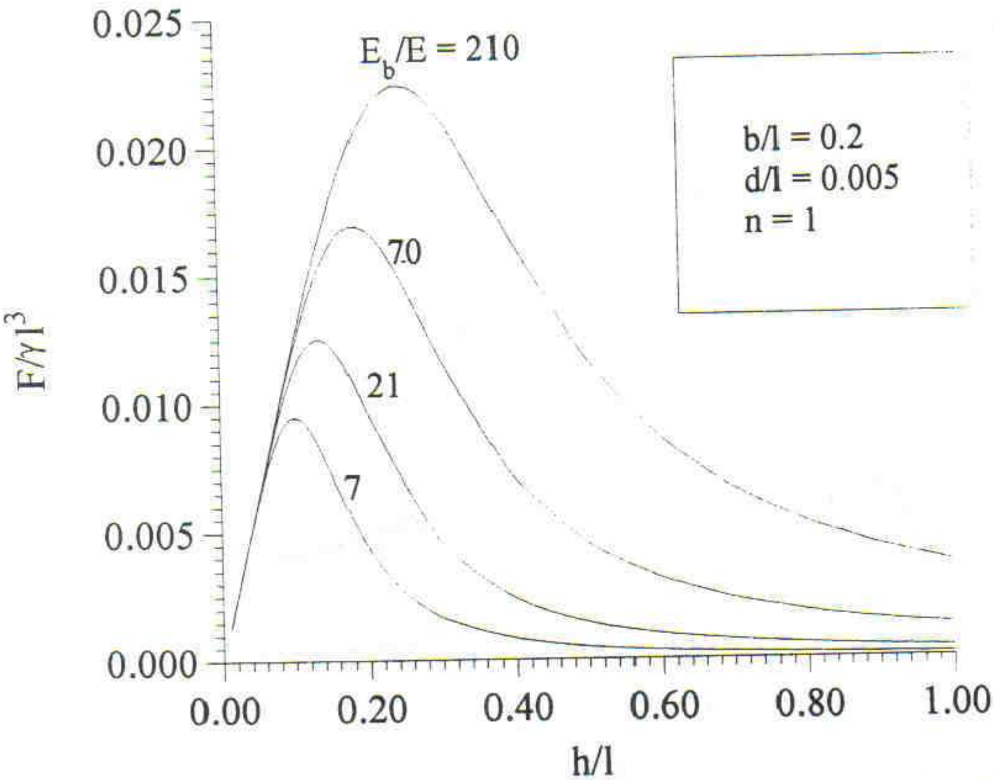

Fig. 7 Variation of Maximum Bolt Force with h/l for Different E_b /E

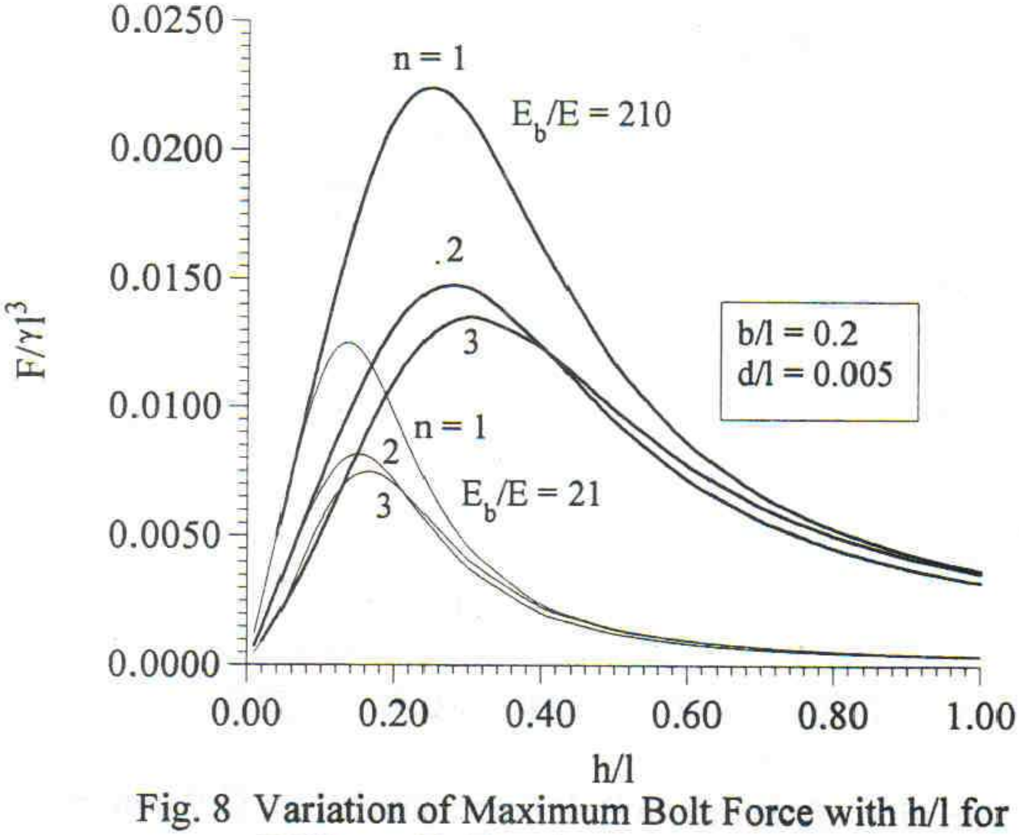

Fig. 8 Variation of Maximum Bolt Force with h/l for Different E_b/E and n

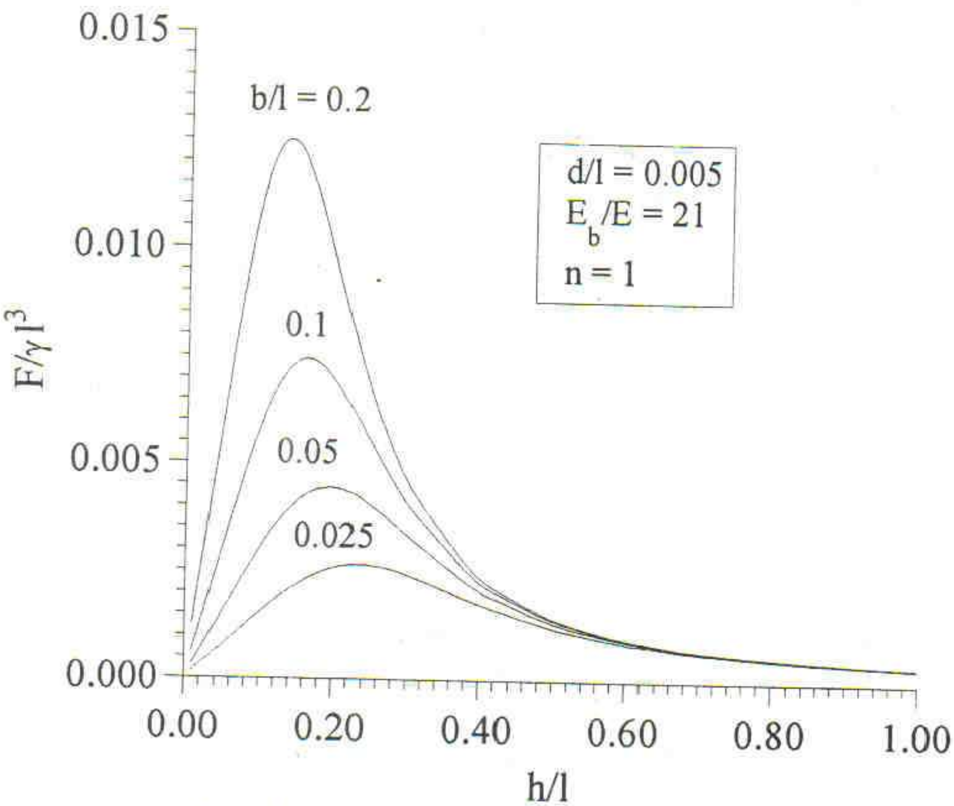

Fig. 9 Variation of Maximum Bolt Force for Different h/l and b/l ratios

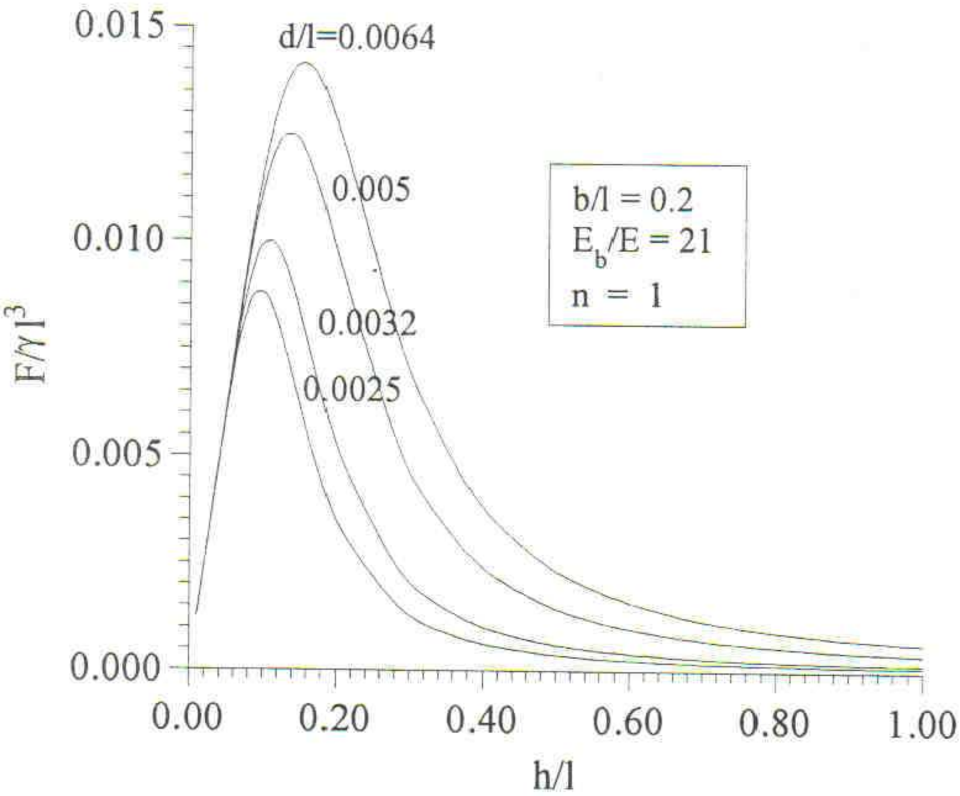

Fig. 10 Variation of Maximum Bolt Force with h/l for Different d/l ratios

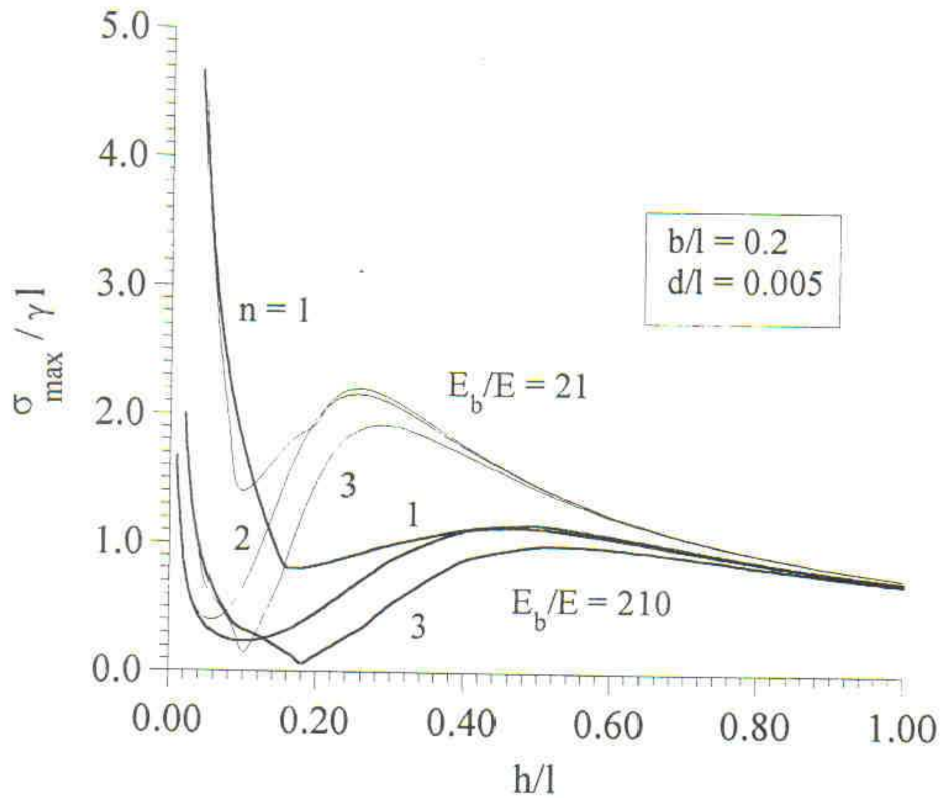

Fig. 11 Effect of n, h/l and E_b/E on Maximum Bending Stresses in Beam