Intelligent Prediction of Elastic Properties of Jointed Rocks

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ABSTRACT

This paper demonstrates the applicability of artificial neural networks in predicting the elastic properties of jointed rocks. The data from uniaxial and triaxial compression tests on different rocks with different joint properties reported in literature was used as input for training the network. A simple method to integrate the properties of joints into single entity called Joint factor (J_f), which can take care of the effects of frequency, orientation and strength of joint as given by Ramamurthy (1994) was used in the analysis. Two different techniques of artificial neural network (ANN) such as Feed Forward Backward Propagation(FFBP) and Radial Basis Function (RBF) networks are used to predict the elastic modulus ratio, E_r. Further two different models 'A' and 'B' were trained to predict the elastic modulus ratio E_r for the unconfined and confined cases respectively. The model 'A' was trained to predict E_r from the Joint factor alone and the model 'B' was trained to predict E_r from J_f and the confining pressure. 90% of the input data was used for the training/learning and the remaining 10% was used for testing the predicting capabilities of the network in both the cases. The results from models were compared with two different regression models from literature. Results from the analysis demonstrate that both the neural network techniques yield similar result and in general neural network approach is efficient in predicting the elastic modulus ratio from joint parameters and confining stress conditions compared to other two regression models tested.

Keywords: Jointed rock, elastic modulus, neural networks, prediction, regression model

1.0 INTRODUCTION

Rock masses are seldom found in nature without joints or discontinuities. Jointed rocks are characterized by the presence of inherent discontinuities of varied sizes with different orientations and intensities, which can have significant effect on their elastic response. Determination of elastic moduli of jointed rocks is very crucial for successful modelling of problems involving jointed rock masses. The modeling of rock masses can be approached in two different ways. In the first approach, the intact rock is modeled using solid elements, whereas the joints are modeled using special joint elements. In the second approach, the rock mass is treated as an equivalent continuum whose properties are assigned in such a way as to represent the contributions of the intact rock and joints towards its overall response. In practice, it is almost impossible to explore all of the joint systems or to investigate all their elastic characteristics and explicitly simulating them in theoretical models. In these cases, the use of the equivalent continuum models to simulate the elastic behaviour of jointed rock masses is found to be promising.

Some simple equivalent continuum models were developed to estimate the strength of rock mass by Zienkiewicz et al. (1977), Gerrard (1982), Hoek and Brown (1997), Barton and Bandis (1990), Ramamurthy (1994), Sridevi and Sitharam (2000) and Sitharam et al., (2001). Majority of these models are developed based on laboratory triaxial testing of small jointed rock samples. The equivalent continuum methods call for the estimation of elastic properties of the jointed rocks concerned. Laboratory determination of the elastic properties of jointed rock cores present practical problem due to large expense and time involved, coupled with the need for highly accurate measurement techniques. Various researchers proposed rock mass classification systems to express the quality and strength of jointed rocks. One of the first such systems to be developed is the Rock Quality Designation (RQD) system by Deere and Deere (1988) to provide a quantitative estimate of rock mass quality from drill core logs. This system only accounts for the frequency of jointing within a rock mass as a measure of its quality. Later systems which have been developed such as the Rock Mass Rating (RMR) by Bieniawski (1973 & 1979) and Q system by Barton (1988) incorporate geological, geometric and design/engineering parameters in arriving at a quantitative value of the rock mass quality.

Another simple method to integrate the properties of joints into single unit is given by Ramamurty (1994) based on extensive laboratory testing of intact and jointed specimens of plaster of paris, sandstone and granite [Arora (1987), Yaji (1984) and Roy (1993)]. This method proposes that a single entity called Joint factor (J_f) can take care of the effects of frequency, orientation and strength of joint. The joint factor J_f is estimated using the following equation

$$J_f = \frac{J_n}{n\,r} \qquad (1)$$

Where, J_n is number of joints per meter depth, 'n' is the inclination parameter depending on the orientation of the joint β, 'r' is the roughness or joint strength parameter depending on the joint condition. The value of 'n' is obtained by taking the ratio of log (strength reduction) at $\beta = 90^o$ to log (strength reduction) at the desired

value of β . This inclination parameter is independent of joint frequency. The values of 'n' are for various orientation angles are given in Table 1 [Ramamurthy (1994)]. The joint strength parameter 'r' is obtained from a shear test along the joint and is given as r = τ_j / σ_{nj} where τ_j is the shear strength along the joint and σ_{nj} is the normal stress on the joint. The values of 'r' are given in Table 2 [Ramamurthy (1994)] based on extensive laboratory testing.

In a field situation joint factor can be estimated from drill core logs. The core size should at least NW size 2.15 inches in diameter and should be drilled with a double–tube core barrel. For the sample thus collected the joint factor can be estimated using equation (1). Number of joints in the meter length of the field specimen gives the value of J_n, the orientation of the joints which are critical for the safety of the structure will determine the value of 'n' (Table 1) and the uniaxial compressive strength of the intact rock will determine the value of 'r' (Table 2). For a given jointed specimen tested in the laboratory, to estimate the joint factor one can calculate the number of joints per meter depth (joint frequency J_n), joint inclination parameter 'n' can be obtained from Table 1 for a given joint inclination, joint strength parameter 'r' can be obtained from Table 2 for given σ_{ci}.

Table 1 - Joint inclination parameter 'n' for different β
[Ramamurthy (1994)]

Orientation of joint β in degrees	Joint inclination parameter 'n'
0	0.82
10	0.46
20	0.11
30	0.05
40	0.07
50	0.31
60	0.46
70	0.63
80	0.82
90	1.00

The equivalent continuum models developed by Ramamurthy (1994) and Sitharam et al. (2001) use joint factor along with intact rock properties to represent the elastic properties of the jointed rocks using a set of statistical relations. Ramamurthy (1994) proposed an exponential model to express the elastic modulus ratio, E_r, which is defined as the elastic modulus of the jointed rock to the elastic modulus of the intact rock, in terms of a joint factor, J_f, and the confining stress, σ_3. Sitharam et al. (2001) expressed E_r in terms of the joint factor alone, in an exponential model, with different relations for different confining stresses. These equations are very simple, involve very less number of input parameters. Both these models are found to be applicable for field problems involving jointed rock masses [(Varadarajan et al. (2001), Sridevi (2000) and Sitharam et al. (2001)].

Table 2 - Values of 'r' for different values of σ_{ci} [Ramamurthy (1994)]

Uniaxial compressive strength of intact rock, σ_{ci} (MPa)	Joint strength parameter, r
2.5	0.30
5.0	0.45
15.0	0.60
25.0	0.70
45.0	0.80
65.0	0.90
100.0	1.00

Though the statistical relations are good enough to predict the properties fairly well, they are limited by the degree of non-linearity they can model. Moreover, statistical relations constrain the data along a particular geometry, which may not always be favorable to capture the non-linear relations existing between various parameters. Artificial Neural Networks have been found to be very efficient in handling non-linear relationships and intelligent prediction of the required parameters. They offer new and exciting possibilities in fields such as pattern recognition where traditional computational methods have not been so successful. Standard computing methods rely on a linear approach to solve problems, while neural networks use a parallel approach similar to the workings of the brain. ANNs have been used for a wide variety of applications in Geomechanics and Rock Engineering [Toll (1996)].

The present paper uses Artificial Neural Network (ANN) approach for the efficient prediction of the elastic moduli of jointed rocks from the confining pressure and the joint factor. A database of results from laboratory triaxial tests on different rocks with different properties of joints was used for training and testing of the network model. The complete database comprised of 523 datasets, of which 237 datasets pertained to unconfined tests and remaining 286 datasets pertained to tests under various confining pressures. The database includes the results from triaxial tests on jointed rocks reported by Arora (1987), Yaji (1984), Roy (1993), Brown and Trollope (1970), and Einstein and Hirschfield (1973). The two ANN techniques such as FFBP and RBF networks are used. The results obtained were compared with those predicted using statistical relations developed by Ramamurthy (1994) and Sitharam et al. (2001). The model was trained to determine the modulus ratio for jointed rock from the joint factor and confining pressure. The aim is to arrive at an efficient system to predict the elastic modulus of jointed rock without predefining a mathematical geometry to correlate the properties. This classifies the problem as one of pattern recognition rather than a conventional trend setting one. Usually, the choice of data for the training and testing datasets is done on a random basis. However, the way the data is divided has a significant effect on the performance of the ANN Model [Shahin et al. (2000)]. The present paper uses a simple data processing method to ensure statistical consistency within the training and testing datasets. The results of the analysis using the ANN model were compared with those obtained from equations proposed by Ramamurthy (1994) and Sitharam et al. (2001). Finally, some factors, which are important while designing any rock engineering problems using ANNs, are discussed.

2. BACKGROUND

Various researchers have established statistical relations to express the elastic modulus of jointed rocks in terms of the joint parameters, collectively expressed in the form of joint factor, and the elastic moduli of the corresponding intact rocks. Ramamurthy (1994) provided exponential relations to express the elastic modulus ratio, E_r, in terms of a joint factor, J_f, and the confining stress, σ_3. These relations were based on laboratory studies on numerous artificial joints. The equation for the unconfined condition was given as follows:

$$E_r = \frac{E_j(\sigma_3 = 0)}{E_i(\sigma_3 = 0)} = \exp\left(-1.15\times10^{-2}\, J_f\right) \tag{2}$$

Tangent elastic modulus of jointed rock for any other σ_3 is derived from the tangent elastic modulus of jointed rock at $\sigma_3 = 0.0$ using the formula given below.

$$\frac{E_j\,(\sigma_3 = 0)}{E_j\,\sigma_3} = 1 - \exp\left[-0.1\left(\frac{\sigma_{cj}}{\sigma_3}\right)\right] \tag{3}$$

Where σ_{cj} is the uniaxial compressive strength of jointed rock and is given as follows:

$$\sigma_{cr} = \frac{\sigma_{cj}}{\sigma_{ci}} = \exp\left(-0.008\, J_f\right) \tag{4}$$

Where σ_{ci} is the uniaxial compressive strength of intact rock.

Equation 1 is valid for $\sigma_3 = 0.0$. For different confining pressures the elastic modulus of jointed rock is calculated using Eq. 3, where σ_{cj} is obtained from Eq. 4 and E_j ($\sigma_3 = 0.0$) is obtained from Eq. 2.

The set of statistical relations given by Sitharam et al. (2001) for estimating the uniaxial compressive strength and elastic modulus of jointed rock are as given below.

$$\sigma_{cr} = \frac{\sigma_{cj}}{\sigma_{ci}} = \exp\left(-0.0065\, J_f\right) \tag{5}$$

$$E_r = \frac{E_j}{E_i} = exp\left(a \times J_f\right) \tag{6}$$

Where ‘a’ is a coefficient determined from the statistical curve fit analyses for the experimental data and J_f is the joint factor. The value of empirical constant ‘a’ for different confining pressures is given in Table 3. For confining pressures other than those listed in Table 3, E_r could be interpolated or extrapolated.

Table 3 - Value of the constant 'a' for different confining pressures [Sitharam et al. (2001)]

Confining pressure (MPa)	Value of 'a'
0.0	-0.0113
1.0	-0.0064
5.0	- 0.0103
7.0	- 0.0082

The relations, thus developed, by Ramamurthy (1994) and Sitharam et al. (2001) were found to be fitting well to their data sets used for testing the relations. However, statistical analyses suffer from some major limitations. For a given database, there is a limit over the regression of the data, which a trend line can accommodate. For a particular database the relation having the maximum possible regression coefficient is ranked highest and the accuracy is considered best. In such a case, the possibility of greater accuracy in prediction and conformity to a wider range of data is completely negated. Moreover, in evolving trend-fitting curves by statistical regression, the data is constrained along a particular two-dimensional geometry. For a dataset containing sparse data, statistical methods encourage the filtering of data outlying a particular geometrical band, to obtain best fitting. In doing so, the essence of the remaining data, and the information contained therein, is completely eliminated. In this light, there is a need for a method of prediction, which can capture the maximum possible information from the whole dataset without confining the correlation along a particular geometry. Artificial Neural Networks have been found to carry out such correlations with great efficiency. They exhibit the capability to analyze highly non-linear relationships existing between various parameters

3. ARTIFICIAL NEURAL NETWORKS

Artificial Neural Networks are computational models in which the mode of data processing replicates the mode of synaptic dynamics in the biological neural networks. The similarity between artificial and biological neural networks lie in the basic parallel and distributive mode of operation, although the later are much more complex. ANNs consist of closely interconnected numerical processing units, which offer a rich structure exhibiting some properties of biological neural networks [Yegnanarayana (1999)]. Evidently, an ANN consists of a large number of interconnections, between its processing units, which carry the weights of the network. The processing units, or neurons, are present in a layered structure comprising of an input layer, an output layer and a hidden layer. A typical three layered feed forward back propagation network is shown in Figure 1a.

Radial Basis Network particularly Generalized Regression Neural Network (GRNN) is also used to perform function approximation task. Figure 1b shows a typical RBF network. Although both RBF and Multi-layer Perceptron such as FFBP are nonlinear feed forward backpropagation networks and are universal approximator fundamentally. They are quite different from each other in their structure. Generally, Feed forward back propagation has less number of neurons in its hidden layer whereas in a RBF

network may have the number of neurons as many as the number of pattern available for training.

3.1 Learning of ANN

Feed Forward Back Propagation (FFBP) Technique

FFBP learning is achieved by training the network pattern wise, i.e. each input is presented to the network one by one and when all the input is presented it is called one epoch. The training of a FFBP network involves a procedure to arrive at a final set of weights in the connections based on which predictions can be made. The Error Back propagation algorithm is the most commonly used algorithm for ANN predictions. The gradient descent algorithm was used to determine the weight updation after each epoch.

The connections between the neuron layers hold the weights in a matrix known as the weight space. Initially, a random set of weights is assigned to the weight space, which is to be optimised by the optimisation algorithm. In a general training scheme, the whole dataset containing sets of input-output pairs is fed to the ANN. The ANN predicts the outputs for the given inputs using the initialised set of weights. The outputs, thus predicted, are compared against the standard output and the sum-squared error is determined. The change in the weight space in the next step depends on this error. It is expressed as the gradient of descent of the error towards the minimum. The change in the weight, after each epoch, for the connection between the i_{th} neuron of one layer and the j_{th} neuron of the next layer is given as

$$\Delta\Psi_{ij} = - \eta \, (\partial \xi / \partial\Psi_{ij}) \qquad (7)$$

Where, η is the learning rate parameter, and Ψ_{ij} is the corresponding weight of the connection. The update of weights for the $(n+1)^{th}$ pattern is given as

$$\Psi_{ij}(n+1) = \Psi_{ij}(n) + \Delta\Psi_{ij}(n) \qquad (8)$$

The error ξ, is the Sum Squared Error and is determined by the following relation

$$\xi = \Sigma \, [\, \Pi'^{\Pi}{}_{k}(n) - \Pi_{k}(n) \,]^2 \qquad (9)$$

Where Π_k (n) is the output determined by the network for the nth pattern and $\Pi'_k(n)$ is the corresponding output given in the training data set.

The input and the hidden layers consist of linear processing units where as the output layer consists of the non-linear processing units. The non-linearity in the processing units in the output layer is achieved using the non-linear Sigmoid function as follows [Zurada, (1997)].

$$f\,(\text{sum}) = 1/ (1 + e^{-(\text{sum})}) \qquad (10)$$

Where (sum) is the weighted sum of the inputs for a processing unit.

The dynamic optimization of the weights continues till a user specified goal is attained. The goal can be specified either as a minimum error goal or as a maximum number of epochs. In the present case, the networks were trained to 5000 epochs.

Radial Basis Network (RBF)

Radial basis networks are based on sound mathematical concepts like theory of interpolation, regularization, kernel regression etc. In this paper, Generalized Regression Neural Network (GRNN) is used which is based on kernel regression. Consider the nonlinear regression model as:

$$y_i = f(x_i) + \varepsilon_i \qquad i = 1,2,.....N \tag{11}$$

x and y are model input and output respectively having 'N' number of patterns. 'N' should be large enough. Here, $f(x_i)$ represent a regression function and ε_i measure of noise in data.

$f(x_i)$ is conditional mean of y for a given x i.e. regression of y on x.

$$f(x) = \int_{-\infty}^{+\infty} y f_y(y|x) dy$$

Where,

$$f_y(y|x) = f_{x,y}(x,y) / f_x(x)$$

$$f(x) = \frac{\int_{-\infty}^{\infty} y\, f_{x,y}(x,y) dy}{f_x(x)}$$

$f_y(y|x)$ is Conditional Probability Density function (pdf) of y, given x. $f_x(x)$ is pdf of x. $f_{x,y}(x,y)$ is Joint pdf of x and y.

To estimate probability density functions, a non parametric estimator Parzen-Rosenblatt density estimator is used, which is as follows (Haykin, 2001),

$$f_x(x) = (1/Nh^m) \sum_{i=1}^{N} K\left(\frac{(x - x_i)}{h}\right) \tag{12}$$

$$f_{x,y}(x,y) = (1/Nh^{(m+1)}) \sum_{i=1}^{N} K\left(\frac{x - x_i}{h}\right) K\left(\frac{y - y_i}{h}\right) \tag{13}$$

Where 'm' is the dimension of input vector. In light of Nadaraya Watson regression estimator approximating function $f(x)$ can be written as:

$$f(x) = \sum_{i=1}^{N} W_{N,i}(x)\, y_i \tag{14}$$

$$W_{N,i}(x) = \left(\frac{K\left(\frac{x - x_i}{h}\right)}{\sum K\left(\frac{x - x_i}{h}\right)} \right) \qquad i = 1,2 \ldots\ldots\ldots\ldots N \tag{15}$$

$K(x)$ is a kernel function. Here, $K(x)$ is chosen as Multivariate Gaussian distribution. $f(x)$ will take the form of

$$f(x) = \frac{\sum_{i=1}^{N} y_i \exp\left(\frac{-\|x - x_i\|^2}{2h^2}\right)}{\sum_{i=1}^{N} \exp\left(\frac{-\|x - x_i\|^2}{2h^2}\right)} \tag{16}$$

In the above equations 'h' is smoothing parameter which is a positive number, this is also called bandwidth as it controls the size of kernel.

Learning of such GRNN is based on estimation of bandwidth 'h' which can be determined by using a suitable generalized cross validation procedure. From equation 16, it is clear that observable values (y_i) can be viewed as weights. All the patterns are used to find out the approximating function. The efficient GRNN can handle if our data is noisy or have many copies of a similar data.

3.2 ANN Model

Two models 'A' and 'B', are prepared for the unconfined and confined cases respectively. The model 'A' was trained to predict the elastic modulus ratio, E_r, from the Joint factor, J_f. Both the networks were trained using the feed forward back propagation algorithm. The model 'A' is also trained by using RBF network. The network B was trained to predict E_r from J_f and the confining stress, σ_3. The basic architecture and the training parameters of the ANNs are shown in Table 4.

Table 4 - Architecture of the FFBP Model

Parameter	Model A	Model B
Number of layers	3	3
Number of neurons in Input	1	2

Number of neurons in Hidden	25	21
Number of neurons in output	1	1
Number of epochs run	5000	5000
Size of training dataset	218	261
Size of testing dataset	23	25

Data Processing for ANN

Shahin et al (2000) reported the relationship between the statistical properties of the training and testing datasets and the model performance. They have also shown that the performance of an ANN model is affected by the proportion of the data chosen to form the testing and training datasets. It is essential to consider the statistical properties, like the mean and standard deviation, of the training and testing datasets to ensure that each dataset represents the same population. The statistical method of division presents a very simple yet effective mode of data division. The data was divided into training and testing datasets using sorting methods, to maintain statistical consistency. In the present case, 10% of the data formed the testing database for each ANN. Consequently, data for the testing datasets were extracted at regular intervals from the sorted database and the remaining 90% of the data formed the training database. The statistical consistency of the data used for the datasets was verified with respect to the mean, standard deviation, maximum value and minimum value. The results of the statistical analyses for unconfined and confined conditions are shown in Tables 5 and 6 respectively.

4. ANALYSIS

The data used in the analyses pertained to laboratory tests conducted by various researchers over a wide range of rock types. The rock types were namely, plaster of Paris, gypsum plaster, sandstone and granite. The intact rock properties of the four rocks are shown in Table 7.

Table 5 - Results from statistical analysis of data used for datasets for unconfined case

Parameter	Dataset	Mean	Median	Standard deviation	Maximum value	Minimum value
J_f	Training	154.5	87.38	156.3	795.8	13
	Testing	185.2	97.56	198.1	778	15.97
E_r	Training	0.3637	0.335	0.269	0.99	0.002
	Testing	0.3571	0.35	0.2961	0.96	0.01

Table 6 - Results from statistical analysis of data used for datasets for confined case

Parameter	Dataset	Mean	Median	Standard deviation	Maximum value	Minimum value
J_f	Training	150.650	87.22	148.195	716.25	13
	Testing	113.433	77.5	92.012	358	19
σ_3	Training	4.338	5	2.452	7	1
	Testing	4.5	5	2.589	7	1
E_r	Training	0.373	0.35	0.272	0.99	0.005

Testing	0.336	0.37	0.226	0.98	0.04

Table 7 - Properties of the intact rock

Property	Sandstone	Agra sandstone	Granite	Gypsum plaster
Mass density (KN/m^3)	22.5	22.17	26.5	15.68
Uniaxial compressive strength (MN/m^2)	70	110	123	20 -50
Cohesion (MN/m^2)	14.0	19.22	25.5	2.17
Angle of internal friction (degrees)	44	51	46.5	34.5

The complete database constituted of 527 datasets, of which 241 pertained to unconfined tests and remaining 286 datasets pertained to varying degrees of confining conditions. The analyses were carried out over confining stresses of 1, 5 and 7 MPa. Consequently, two groups, A and B, corresponding to the unconfined and confined cases respectively, were identified and analyzed separately. In the group A, 218 datasets were used to develop the various models, and hence formed the training set, and the remaining 23 datasets were used to evaluate the performance of the models, and hence formed the testing dataset. In the group B, 261 datasets constituted the training set and 25 sets constituted the testing set. The division of the data into the testing and training datasets was done on a regular sampling basis, in order to maintain statistical consistency. This helped in maintaining a uniform distribution of the data in each set throughout the whole sample space. This method of data division was meant to serve a dual purpose, as it has been observed that statistical consistency of the training and testing data enhances the performance of an ANN and subsequently helps in evaluating them better (Shahin et al., 2000).

The datasets were analyzed using three different models, which use the joint factor (J_f) to represent the joint parameters and predict the elastic modulus ratio (E_r). These models are the ANN models and the exponential models given by Ramamurthy (1994) and Sitharam et al. (2001) from regression analysis of experimental data. The performance of the ANN models as developed in the present work was compared against the remaining two models. As mentioned earlier, the training data sets were used for predicting E_r from the models. These models were evaluated and compared on the basis of their accuracies of predictions over the testing datasets. In the confined case, the joint factor was the only input for each of the models. The outputs, E_r, of the models were compared against the standard values of the testing dataset. In the confined case the inputs were the Joint factor (J_f) and the Confining stresses (σ_3).

5. RESULTS AND DISCUSSION

Neural network modeling of rock joint properties yielded some interesting results. The results from both FFBP and RBF are presented in this section. The results from the analyses are compared with the results from analyses using regression models for unconfined as well as confined cases.

5.1 Case A:

Case A deals with the prediction of modulus ratio (E_r) values from the joint factor (J_f) for the unconfined case using both networks with model 'A' and the equivalent continuum models developed by Ramamurthy (1994) and Sitharam et al. (2001). The predicted values of modulus ratio E_r using ANN for the unconfined case are plotted against the original experimental values of E_r, in the testing dataset in Fig. 2. The accuracy of prediction is estimated by drawing a 45° line to represent the 100% accuracy in prediction. A visual inspection reveals that, of the 23 predicted values of E_r, 7 values are very close to the 45° line, which represents the pattern of original experimental values, 11 values lie above the line and remaining 5 values lie below the line for FFBP network and for RBF network 6 values lie on or very close to 45 degree line, 9 values lie above the line and 8 values lie below the line. The E_r values predicted by the regression models of Ramamurthy (1994) and Sitharam et al. (2001) for the same values of J_f are given in Figs. 3 and 4 respectively. From Fig. 4 it can be observed that out of the 23 predicted E_r values by Ramamurthy's (1994) model, only three values lie on the 45° line,10 values lie below the line and remaining 10 values lie above the line. Figure 4 shows that the E_r values predicted by Sitharam et al. (2001) follow almost identical trend as predicted by Ramamurthy (1994) for the unconfined case. It can be seen that the two regression models define an upper bound prediction pattern for the data as values below the 45 degree line are sparse. Whereas the data predicted by the neural network is in close range with the experimental data. This can be attributed to the fact that the predicted data in this case is not governed by any particular geometry. And hence, it has a fair probability to follow the non-linear pattern of the original data. The values predicted by FFBP is very close to that of predicted by RBF network for unconfined case. This aspect is illustrated in Fig. 5, where the experimental and predicted E_r values are plotted against the joint factor (J_f). This figure clearly shows how the ANN prediction is taking care of the non-linearity in the data compared to the other two exponential models. Clearly, from the plots, all the methods have comparable degrees of accuracy, so that the efficiency of one method over the other cannot be well appreciated.

Table 8 shows the percentage error of prediction obtained using each of the methods for the unconfined case. Clearly, the mean accuracy of prediction using ANNs is higher as compared to the other two models. But that cannot be taken as the sole evidence for the better predicting capabilities of the ANNs over statistical methods. The accuracy of the models has to be tested for the confined case also to judge the predicting capabilities of these models.

Table 8 - Percentage error in predicting E_r using different models for unconfined case

	ANN Model		Ramamurthy	Sitharam et al.
	FFBP	RBF	(1994)	(2001)
Average Absolute percentage error	38.907	38.33	45.388	45.734
Average accuracy	0.6109	0.616	0.546	0.543

5.2 Case B:

Figure 5 clearly shows that 'Er' values predicted by FFBP is very close to those predicted by RBF network, so case B is analyzed by FFBP only. Case B deals with the prediction of E_r values from the joint factor and confining pressure using FFBP network for model 'B' and the other two regression models. The predicted values of modulus ratio E_r using FFBP for the confined case are plotted against the original experimental values of E_r, in the testing dataset in Fig. 6. From figure, it can be observed that out of 25 predicted values of E_r, 4 values lie very close to 45° line, 10 values lie above the line and remaining 11 values lie below the line. But almost all the predicted values lie fairly close to the line, indicating that the values follow the experimental trend line. The E_r values predicted by the regression models of Ramamurthy (1994) and Sitharam et al. (2001) for the same values of J_f are given in Figs. 7 and 8 respectively. From Fig. 7 it can be observed that out of the 25 predicted E_r values by Ramamurthy's (1994) model, three values lie on the 45° line, 7 values lie above the line and remaining 15 values lie below the line. Fig. 8 shows that in case of E_r values predicted by Sitharam et al. (2001), three values lie close to the 45° line, 10 values lie above the line and remaining 12 values lie below the line.

In the confined case the difference between the predicting capabilities of the models is very apparent from the plots. It can be seen from Figs. 7 and 8 that about 25% of the data is scattered much away from the 45° line, which clearly shows that the regression models fail in these particular cases. Where as from Fig. 6, it is evident that all the data predicted by FFBP model lies fairly close to the 45° line indicating that in all the cases the prediction is following the experimental trend. The predictions of FFBP and the other two regression models for the confined case are compared by plotting the E_r values against J_f values in Figs. 9 and 10. The predictions of the regression models follow different exponential trends for different confining pressures. The effect of confining pressure is inherently taken care of by FFBP model and hence there is a single curve representing the prediction for all confining pressures. From these figures, it is obvious that the regression models are unable to model the non-linearity inherent in the distribution of the data for the confined case, particularly when the value of the joint factor is high. For the joint factor value up to 200, the FFBP prediction is falling in between the series of predicted curves for both the regression models. For higher J_f values, FFBP prediction is falling out of range of the band formed by the exponential curves and is following the experimental trend more closely. This is clear evidence for the better predicting capabilities of the FFBP for the confined case compared to the other two models tested. The reduction in accuracy of the regression models for the confined case can be attributed to the inherent deficiency of regression equations in correlating more than two parameters. However, with the ANN network the parameters are not constrained. The correlation is carried out according to the independent behaviour of each output parameter with respect to each of the input parameter. This enhances the correlations and hence the efficiency of predictions.

The percentage error in prediction and mean accuracy of the various models for the confined case are shown in Table 9. Clearly the accuracies obtained using ANN model are much higher as compare to the other statistical methods in this case.

Table 9 - Percentage error in predicting E_r using different models for confined case

	FFBP	Ramamurthy (1994)	Sitharam et al. (2001)
Average absolute percentage error	18	31	34
Average accuracy	0.82	0.69	0.66

6. CONCLUSIONS

In the present study, Artificial Neural Networks have been used for the efficient prediction of the elastic modulus ratio from joint parameters and confining stress conditions. The relations given by Ramamurthy (1994) and Sitharam et al. (2001) have been found to give satisfactory results for predictions in the unconfined case. However, it has been observed that these relations like give an upper bound prediction of data, which may not always be acceptable, even though the average accuracy is satisfactory.

The efficiency of ANN model is much evident for confined case, for which the values predicted by the regression models greatly differ from the experimental values in some instances. This reflects that statistical relations are inherently incapable to model the non-linearity existing within the data. On the other hand, the Artificial Neural Networks, because of their rich structures for correlations, have been found to capture this non-linearity with comparable degree of accuracy above the tested regression models. The prediction curve for ANN model is unique for different confining pressures, unlike the band of exponential curves in case of regression models. The ANN prediction falls within the band for joint factor values less than 200 and falls out of the band for higher J_f values, following the experimental trend more closely for all the values of joint factor. Thus the ANN model provides significant advantage for handling problems involving practical discontinuous system. The present work supports the use of neural networks for the successful prediction of elastic properties of jointed rocks and opens up the possibility of embedding neural networks into numerical modeling codes for modeling the structures in jointed rocks.

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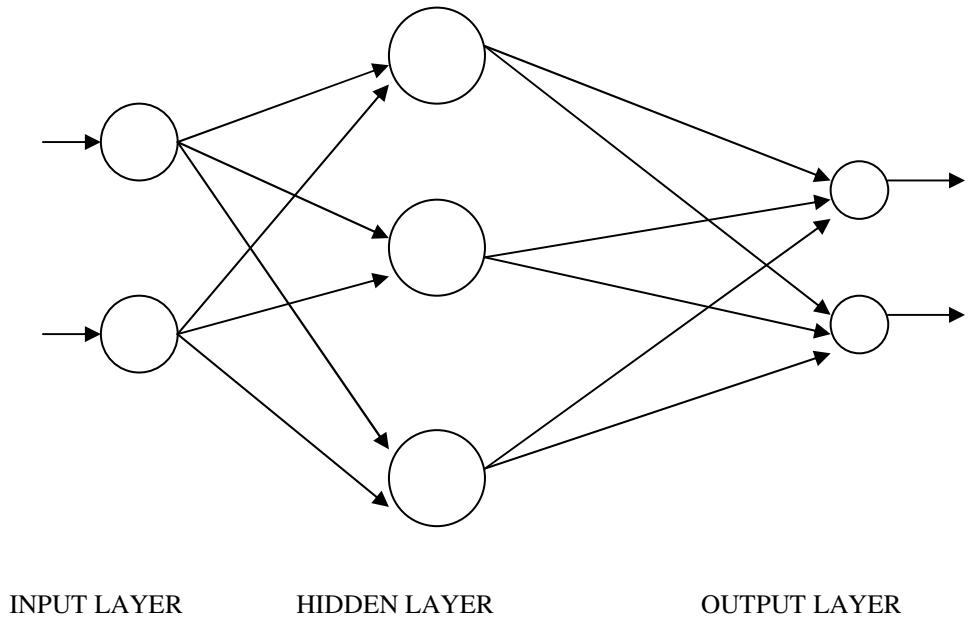

Fig. 1a - Three- layered feed forward back propagation Artificial Neural Network

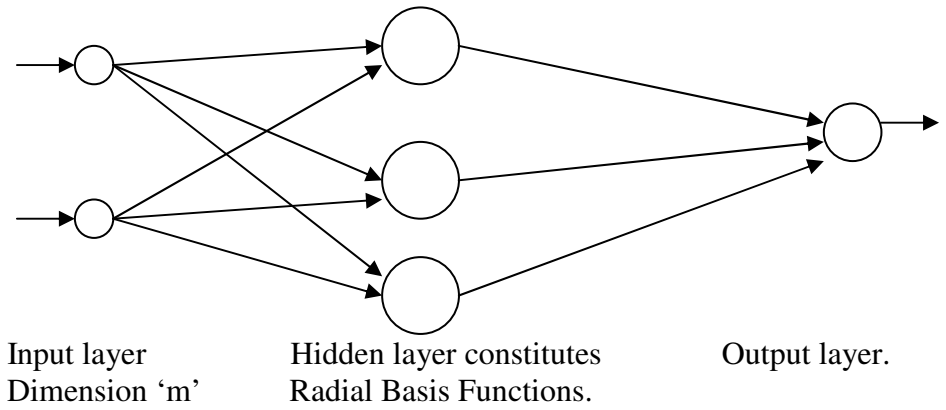

Fig.1b - A typical radial basis function network

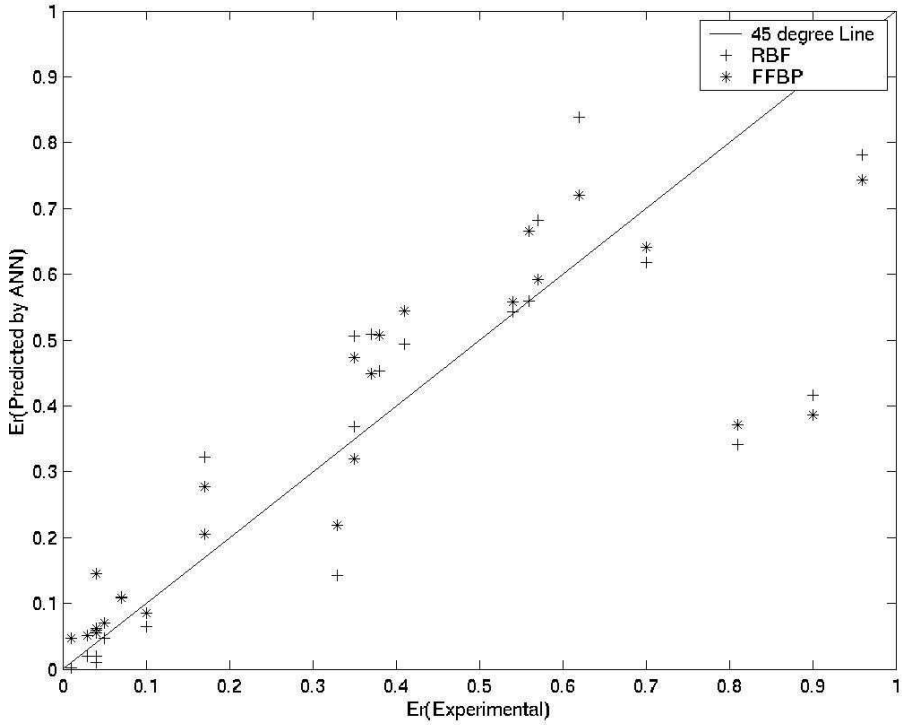

Fig. 2 - Comparison of E_r values predicted by ANN with experimental Values for unconfined case

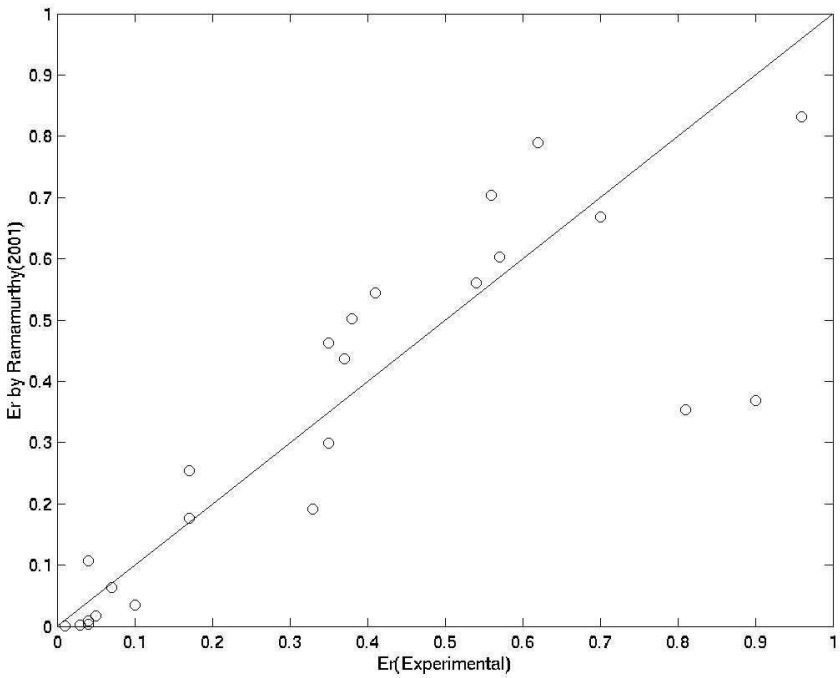

Fig. 3 - Comparison of E_r values predicted by Ramamurthy (1994) with experimental values for unconfined case

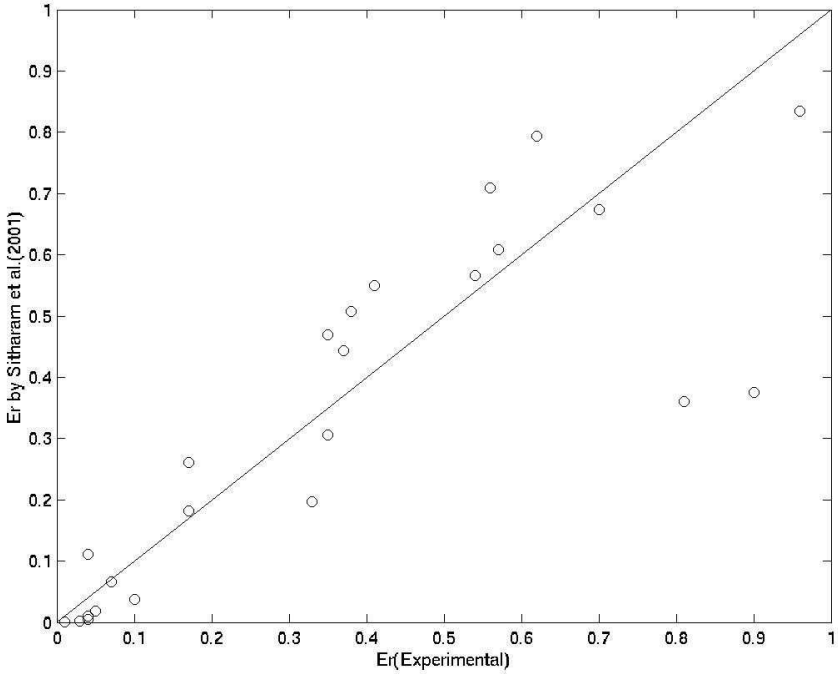

Fig. 4 - Comparison of E_r values predicted by Sitharam et al. (2001) with experimental values for unconfined case

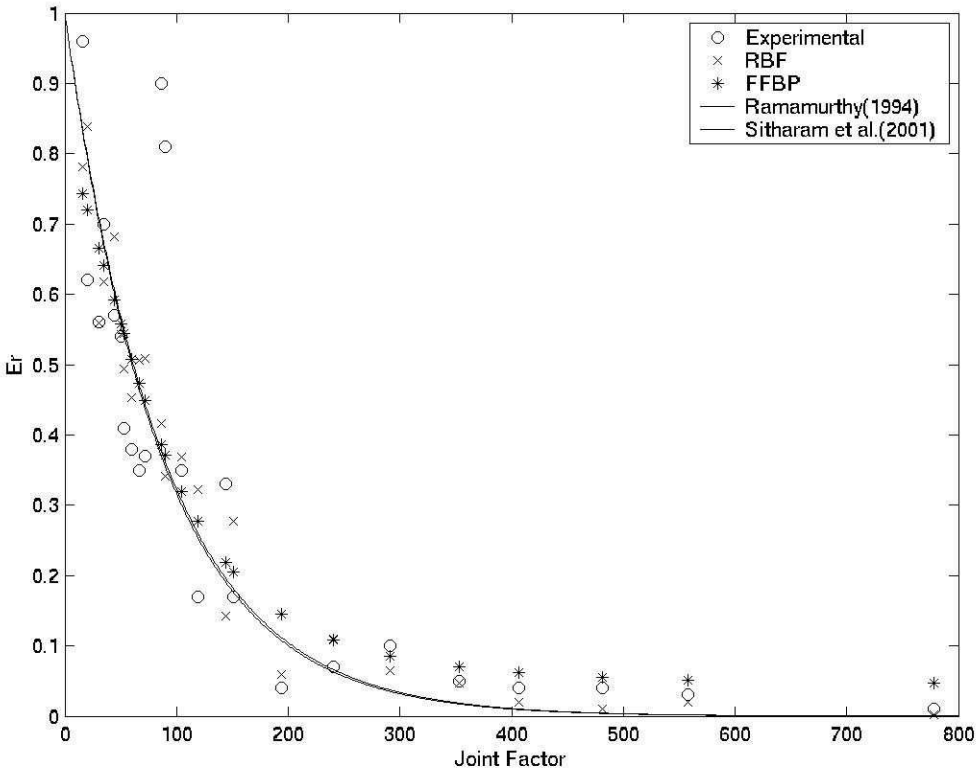

Fig. 5 - Comparison of ANN model with the regression models proposed by Ramamurthy (1994) and Sitharam et al. (2001) for unconfined case

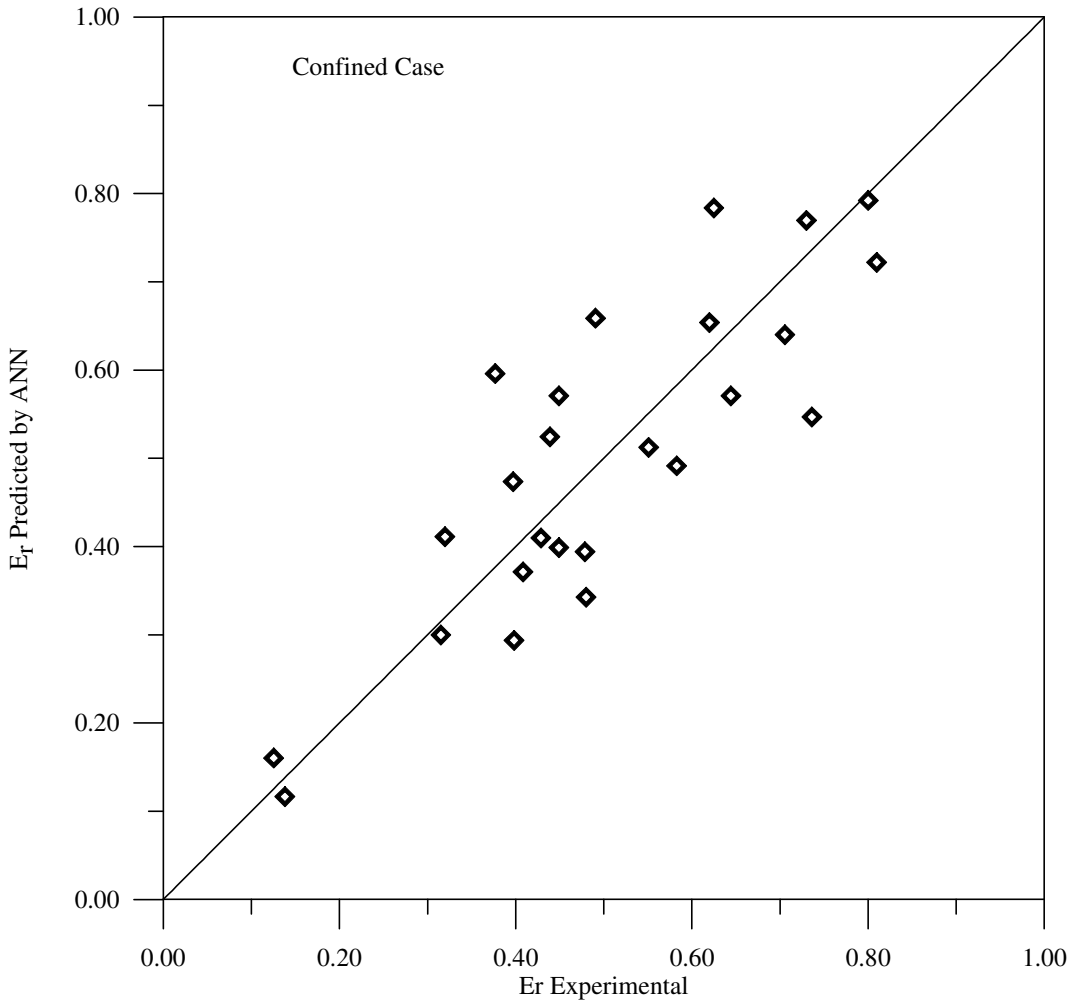

Fig. 6 - Comparison of E_r values predicted by ANN with experimental values for confined case

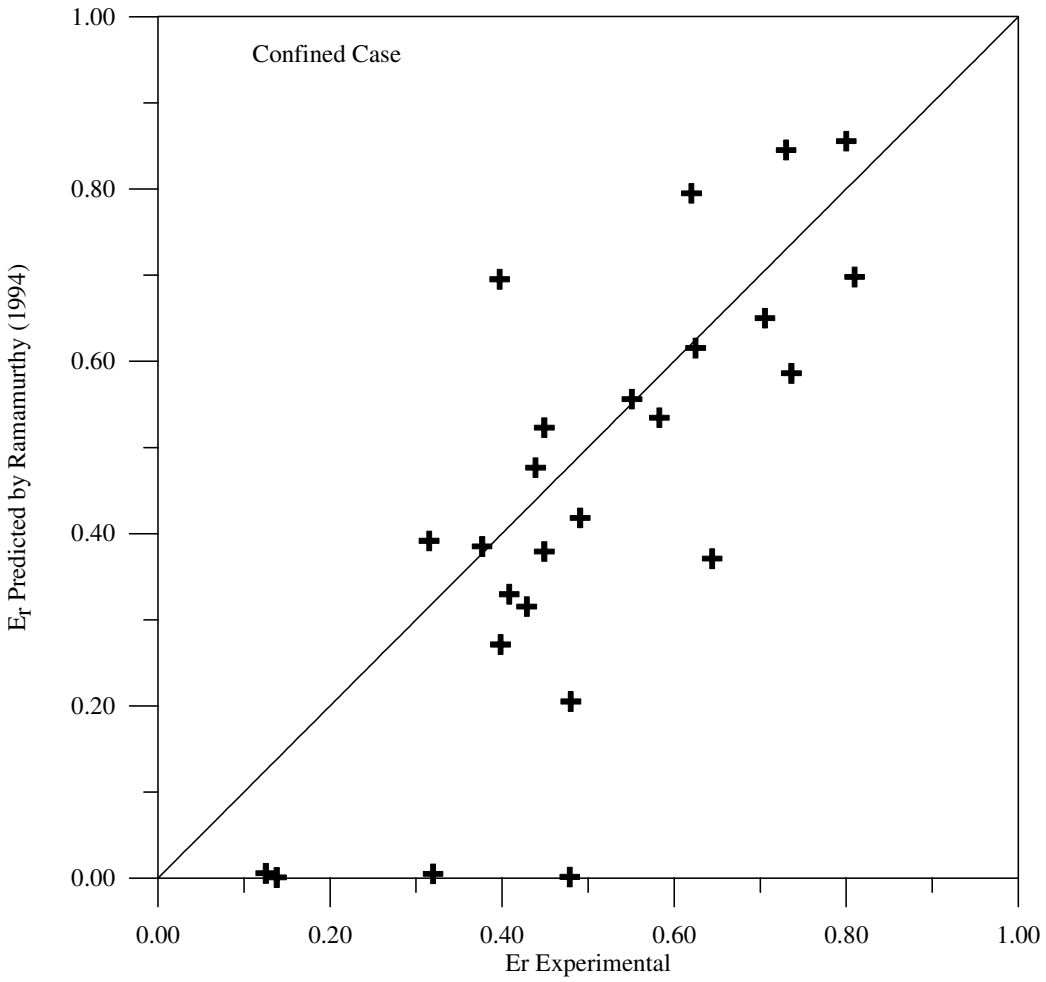

Fig. 7 - Comparison of E_r values predicted by Ramamurthy (1994) with experimental values for confined case

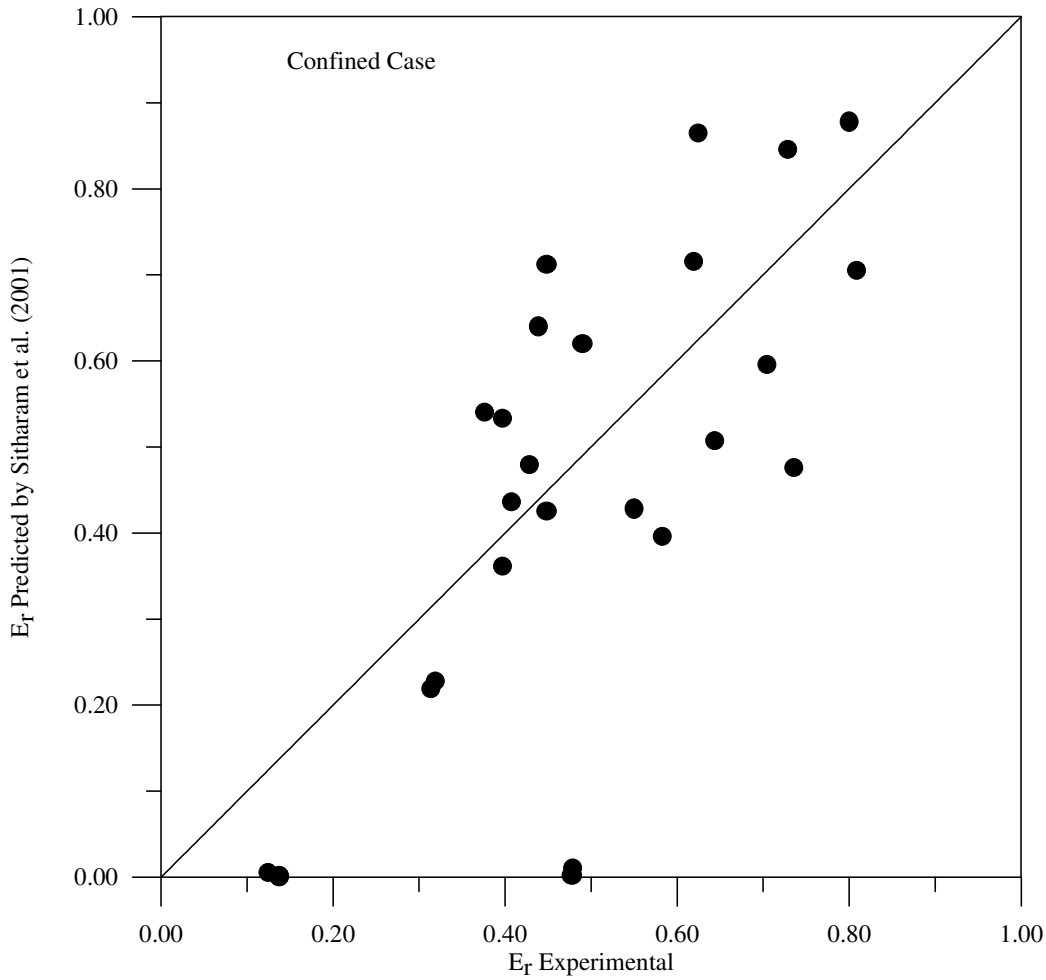

Fig. 8 - Comparison of E_r values predicted by Sitharam et al. (2001) with experimental values for confined case

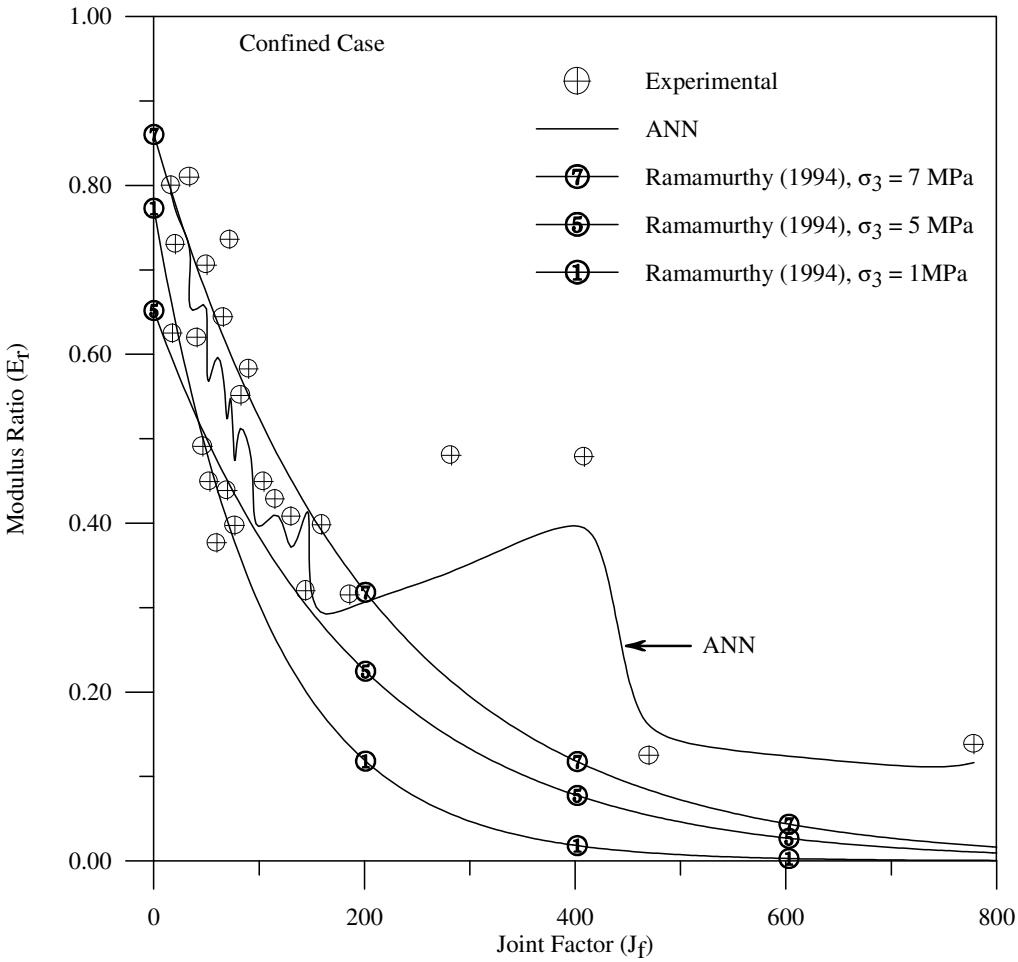

Fig. 9 - Comparison of ANN model with the regression model proposed by Ramamurthy (1994) for confined case

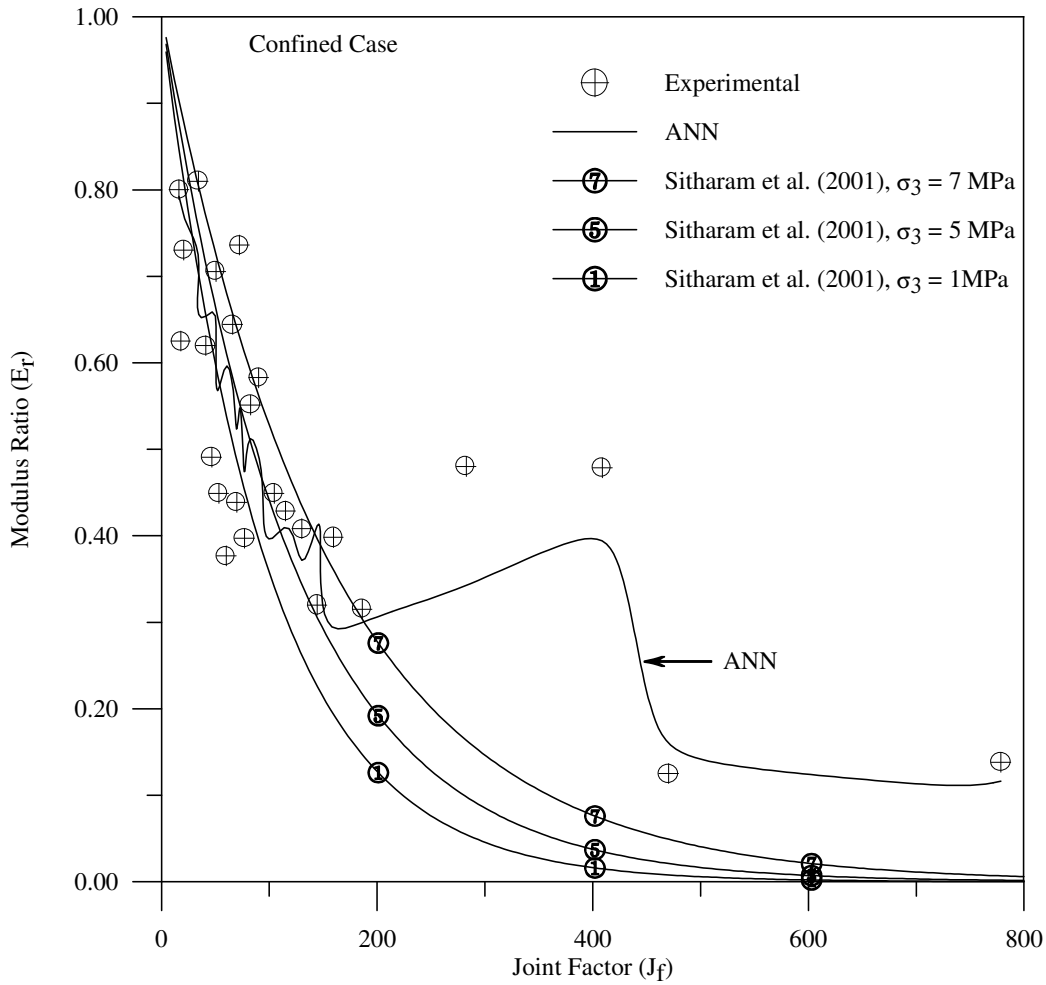

Fig. 10 - Comparison of ANN model with the regression model proposed by Sitharam et al. (2001) for confined case