Prediction of Stress - Strain Behaviour of Intact and Jointed Rocks in Triaxial Compression

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ABSTRACT

A simple statistical relation to obtain the elastic modulus of jointed rocks and thereby predicting their stress-strain response in triaxial compression is presented in this paper. This equation has been arrived from the multi variant regression analysis of large amount of experimental data reported in literature. In this equation, the jointed rock is represented as a continuum material with equivalent elastic modulus ($E_j$) obtained from the properties of the intact rock and joint factor ($J_f$). Joint factor is the integration of the properties of joints to take care of the effects of frequency, orientation and strength of joint. The equation presented in this paper is simple compared to similar equation developed by earlier researchers. The effect of confining pressure is also incorporated in the equation thus facilitating the use of a single equation to use for any confining pressure. This equation is incorporated in a commercial finite difference program Fast Lagrangian Analysis of Continua (FLAC) to carry out the equivalent continuum analysis of jointed rock samples tested in triaxial compression. The constitutive behaviour of the rock is represented by a confining stress dependant hyperbolic relation with Mohr-Coulomb failure criterion. The numerical model has been validated against experimental results from wide range of intact and jointed rock samples with different joint fabric and joint orientation and for a wide range of confining pressures (1 MPa to 155 MPa). Results showed that the stress-strain curves obtained from numerical analysis are matching closely with the experimental stress-strain curves for various intact and jointed rock samples exhibiting linear to highly nonlinear stress-strain behaviour. This study confirmed that the numerical model developed in the present study can efficiently simulate the effects of number of joints, strength of joint, orientation of joint, type of rock and confining pressure.

Keywords: Rocks, intact, jointed, triaxial, stress-strain, equivalent continuum, numerical model
1. INTRODUCTION

Natural discontinuities to some level are common features of rocks. Discontinuities in zones of high stresses near an underground deep excavation can provide planes for shear failure and displacement. All of the currently accepted design methods for foundations, slopes and underground excavations in rock masses require geometric and mechanical information of the discontinuities. Determination of elastic modulus of jointed rocks is very important for successful design of structures involving jointed rock masses. Design methods can be categorized as analytical, observational or empirical. Empirical methods assess the stability of structures by the use of past practices to predict future behaviour based upon factors most critical towards the design. In practice, it is almost impossible to explore all of the joint systems or to investigate all their elastic characteristics and explicitly simulating them in theoretical models. Thus employing empirical derivations to model rock mass as continuum with equivalent material properties has gained acceptance over the last fifteen years. The process that the authors have found to be of greatest value is to employ equivalent continuum approach into numerical codes from a successful tool for the prediction of the behaviour of jointed rock masses.

Some equivalent continuum models were developed to simulate the jointed rock mass by Singh (1973a, 1973b), Zienkiewicz et al. (1977), Gerrard (1982), Cai and Horii (1992), Yoshida and Horii (1998), Oda et al. (1993) and Kawamoto et al. (1988). Some researchers have developed empirical relations to estimate the equivalent material properties of the jointed rock mass from the geometrical and mechanical properties of discontinuities. These equations can be incorporated in other constitutive models for effective simulation of jointed rock masses, e.g. Wei and Hudson (1986) Barton and Bandis (1990), Hoek and Brown (1997), Ramamurthy (1994), Sridevi and Sitharam (2000) and Sitharam et al. (2001a). Most of these models are developed based on laboratory triaxial testing of small jointed rock samples. The equations based on joint factor to estimate the equivalent elastic modulus for jointed rock mass proposed by Ramamurthy (1994) and Sitharam et al. (2001a) from extensive laboratory testing of intact and jointed rock specimens are very simple and practical among all the models stated above. These equations require the estimation of two simple joint parameters, namely the number of joints in the rock per meter depth and the inclination of most critical joint set. Detailed description of the model and design tables for estimating the joint strength parameter from the unconfined compressive strength of the intact rock and joint inclination parameter from the orientation of the joint are given by Ramamurthy (1994) and Sitharam et al. (2001a).

Ramamurthy (1994) developed two different equations for obtaining the modulus ratio of jointed rocks, one equation for the unconfined case representing the uniaxial compression and the other for the confined case, representing the triaxial compression. The equation given for the modulus ratio of jointed rock in triaxial compression calls for the estimation of the
elastic modulus of the jointed rock in uniaxial compression separately, which in turn requires the estimation of the uniaxial compressive strength of jointed rock. This involves total three equations to be used to obtain the elastic modulus of a jointed rock sample in triaxial compression. Sitharam et al. (2001a) developed different equations to obtain the elastic modulus of a sample at different confining pressures in triaxial compression. In the present study, a single regression equation is developed from the multi variant regression analysis of the same data used by Ramamurthy (1994) and Sitharam et al. (2001a), considering the confining stress \( \sigma_3 \) as an extra parameter. This equation is comparatively simple in the sense the effect of confining pressure also is incorporated in the equation thus making it very simple to use for any confining pressure in triaxial compression. This regression equation is incorporated in the commercial finite difference program Fast Lagrangian Analysis of Continua [FLAC] (Cundall, 1976; Itasca, 1995), that is widely used for modeling of rock masses. The triaxial compression tests carried out on different rocks with different joint parameters at different confining pressures are simulated in FLAC, the elastic modulus of the jointed rocks obtained using the new regression equation developed from the present study. The results are presented in the form of stress-strain curves. The results from the numerical analysis are compared with the results from laboratory triaxial compression tests.

2. NEW REGRESSION EQUATION

Various researchers have established statistical relations to express the elastic modulus of jointed rocks in terms of the joint parameters and the elastic moduli of the corresponding intact rocks. Ramamurthy (1994) provided exponential relations to express the elastic modulus of jointed rock in terms of a joint factor, \( J_f \), and the confining stress, \( \sigma_3 \). These relations were developed based on laboratory studies on numerous artificial joints reported by Roy (1993), Arora (1987), Yaji (1984) and Einstein and Hirschfield (1973). Joint factor for a given jointed rock can be estimated based on the joint frequency \( J_n \), orientation of the joints \( \beta \) with respect to the direction of major principal stress and the strength of the joint. The joint factor for given jointed rock is estimated using the following equation:

\[
J_f = \frac{J_n}{n \ r}
\]

Where, \( J_n \) is number of joints per meter depth, ‘n’ is the inclination parameter depending on the orientation of the joint \( \beta \), ‘r’ is the roughness or joint strength parameter depending on the joint condition. The value of ‘n’ is obtained by taking the ratio of log (strength reduction) at \( \beta = 90^\circ \) to log (strength reduction) at the desired value of \( \beta \). This inclination parameter is independent of joint frequency. The values of ‘n’ are for various orientation angles and the joint strength parameter ‘r’ for various uniaxial compressive strengths of intact rock are given by Ramamurthy (1994) based on extensive laboratory testing and are listed in Tables 1 and 2.
Table 1 - Joint inclination parameter ‘n’ for different β [after Ramamurthy (1994)]

<table>
<thead>
<tr>
<th>Orientation of joint β in degrees</th>
<th>Joint inclination parameter ‘n’</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.82</td>
</tr>
<tr>
<td>10</td>
<td>0.46</td>
</tr>
<tr>
<td>20</td>
<td>0.11</td>
</tr>
<tr>
<td>30</td>
<td>0.05</td>
</tr>
<tr>
<td>40</td>
<td>0.07</td>
</tr>
<tr>
<td>50</td>
<td>0.31</td>
</tr>
<tr>
<td>60</td>
<td>0.46</td>
</tr>
<tr>
<td>70</td>
<td>0.63</td>
</tr>
<tr>
<td>80</td>
<td>0.82</td>
</tr>
<tr>
<td>90</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2 - Values of ‘r’ for different values of $\sigma_{ci}$ [after Ramamurthy, (1994)]

<table>
<thead>
<tr>
<th>Uniaxial compressive strength of intact rock, $\sigma_{ci}$ (MPa)</th>
<th>Joint strength parameter, r</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>0.30</td>
</tr>
<tr>
<td>5.0</td>
<td>0.45</td>
</tr>
<tr>
<td>15.0</td>
<td>0.60</td>
</tr>
<tr>
<td>25.0</td>
<td>0.70</td>
</tr>
<tr>
<td>45.0</td>
<td>0.80</td>
</tr>
<tr>
<td>65.0</td>
<td>0.90</td>
</tr>
<tr>
<td>100.0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

The equation for obtaining the elastic modulus of jointed rock for any confining pressure $\sigma_3$ in triaxial compression is given by Ramamurthy (1994) as

$$E_j = \frac{E_j(\sigma_3 = 0)}{1 - \exp\left[-0.1\left(\frac{\sigma_{ci}}{\sigma_3}ight)\right]}$$

(2)

In the above equation, $E_j(\sigma_3=0)$ is the elastic modulus of the jointed rock in uniaxial compression, which can be calculated using the following equation.

$$E_j(\sigma_3 = 0) = \exp\left[-1.15 \times 10^{-2} J_f \right] E_i(\sigma_3 = 0)$$

(3)

Where $E_i(\sigma_3=0)$ is the elastic modulus of the intact rock in uniaxial compression. In equation (1), $\sigma_{cj}$ is the uniaxial compressive strength of jointed rock given as


\[ \sigma_{c_j} = \exp(-0.008 J_f) \sigma_{ci} \quad (4) \]

Where \( \sigma_{ci} \) is the uniaxial compressive strength of intact rock. The set of statistical relations given by Sitharam et al. (2001a) for estimating the elastic modulus of jointed rock at different confining pressures are represented as

\[ E_j = \exp(-a \times J_f) E_i \quad (5) \]

Where ‘a’ is an empirical constant determined from the statistical curve fit analyses for the experimental data and \( J_f \) is the joint factor. The value of ‘a’ for three different confining pressures is given in Table 3. For confining pressures other than those listed in Table 3, \( E_r \) could be interpolated or extrapolated.

<table>
<thead>
<tr>
<th>Table 3 - Value of the constant ‘a’ for different confining pressures [after Sitharam et al. (2001a)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Confining pressure (MPa)</td>
</tr>
<tr>
<td>1.0</td>
</tr>
<tr>
<td>5.0</td>
</tr>
<tr>
<td>7.0</td>
</tr>
</tbody>
</table>

The estimation of elastic modulus of jointed rock using Eq. 5 requires the estimation of the coefficient ‘a’ for different confining pressures. Moreover, the linear interpolation for obtaining ‘a’ for confining pressures other than the three confining pressures is not justified for higher confining pressures, where the variation of elastic modulus with confining pressure is highly nonlinear.

The complexities involved in the estimation of the elastic modulus of the jointed rock in triaxial compression using the above two approaches are eliminated by using multi variant regression, considering the confining pressure (\( \sigma_3 \)) as extra variable to arrive at a single equation. The data used for the regression analysis includes results of experiments on different types of rocks at three different confining pressures 1, 5, 7 MPa taken from Roy (1993), Arora (1987), Yaji (1984), Brown and Trollope (1970) and Einstein and Hirschfield (1973) and is presented in Figs. 1 to 3. The uniaxial compressive strength of different rocks used for the multi variant regression analysis is also given in the figures. To differentiate between the same type of rock samples with different properties, a number is given following their names, e.g. plaster of Paris 1 and plaster of Paris 2. Modulus ratio in the figures is the ratio of the elastic modulus of the jointed rock to the elastic modulus of the intact rock.

The new statistical equation obtained from the multi variant regression analysis of the data is given below.
In the above equation, \( E_j \) is the elastic modulus of the jointed rock at a confining pressure of \( \sigma_3 \), \( \sigma_{ci} \) and \( E_i \) are the uniaxial compressive strength and elastic modulus of the intact rock respectively and \( J_f \) is the joint factor obtained from Eq. 1.

3. NUMERICAL MODEL

Triaxial compression tests on intact and jointed rocks are simulated in numerical analysis using axisymmetric model. In the present study the explicit two-dimensional finite difference program Fast Lagrangian Analysis of Continua [FLAC] version 3.3 [Cundall, (1976); Itasca, (1995)] is used for the analysis. The grid used for simulating the triaxial test sample is shown in Fig. 4. Only symmetric quarter of the total test sample is simulated in the numerical analysis since stress-state will be isotropic because of the equivalent uniform properties assumed for the jointed rock sample. A non-linear elastic confining stress dependant model following hyperbolic relation proposed by Duncan and Chang (1963) with Mohr-Coulomb failure criterion is used in the present study. The material behavior of intact rock is modeled using the following nonlinear relation.

\[
E_i = \left[ 1 - \frac{R_f (1 - \sin \phi) (\sigma_1 - \sigma_3)}{2c \cos \phi + 2 \sigma_3 \sin \phi} \right] \frac{K P_a}{\sigma_3} \left( \frac{\sigma_3}{P_a} \right)^n
\]  

(7)

Where, \( E_i \) is the elastic modulus of the intact rock, \( \sigma_1 \) and \( \sigma_3 \) are the major and minor principal stresses respectively, \( c \) and \( \phi \) are shear strength parameters of intact rock and \( K \) and \( n \) are hyperbolic parameters for the intact rock. \( R_f \) is failure ratio given as

\[
R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}}
\]  

(8)

where \((\sigma_1 - \sigma_3)_f\) is the failure stress and \((\sigma_1 - \sigma_3)_{ult}\) is the ultimate or the asymptotic value of stress.

The numerical analysis was performed in two stages, the 1st stage being the confining pressure application and the 2nd stage being the application of the deviator stress. The analyses were performed using displacement control method in which equal vertical deformation is specified to the grid points on the top surface to simulate the straining of the sample. The vertical deformation was applied by initializing the velocity of the grid points on the top surface to a very small value of 3e-7 m per load step. Loading was done in
several steps with 10 iterations per load step. Equilibrium is defined in the analysis as the state in which the out-of-balance force is less than 100 N.

4. RESULTS AND DISCUSSION

4.1 Numerical Analysis of Intact Rocks

Numerical analysis of the intact rock samples was first taken up. Three different intact rocks for which the experimental stress-strain curves were reported in the literature were selected for the analysis. Kota sandstone with linear stress-strain response (Yaji, 1984), Haizume siltstone with slightly nonlinear stress-strain response (Hoshino et al., 1972) and Yamaguchi marble with highly nonlinear stress-strain response (Gokhale and Ramamurthy, 1981) were selected and their stress-strain responses were predicted from numerical analysis. The properties of these rocks and the confining pressures at which they were tested are given in Table 4.

Table 4 - Properties of rocks used for the numerical analysis of intact rocks

<table>
<thead>
<tr>
<th>Intact Rock Type</th>
<th>Hyperbolic Parameters</th>
<th>Cohesion (c)</th>
<th>Angle of internal friction ($\phi$)</th>
<th>Failure Ratio ($R_f$)</th>
<th>Confining pressure ($\sigma_3$) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kota sandstone</td>
<td>35279 0.113</td>
<td>11</td>
<td>44°</td>
<td>0</td>
<td>1, 2.5, 5</td>
</tr>
<tr>
<td>Haizume Siltstone</td>
<td>1001 0.39</td>
<td>11.3</td>
<td>19°</td>
<td>0.5</td>
<td>20, 50, 100</td>
</tr>
<tr>
<td>Yamaguchi Marble</td>
<td>440986 0.091</td>
<td>40</td>
<td>26.7°</td>
<td>0.99</td>
<td>31, 52, 155</td>
</tr>
</tbody>
</table>

The comparison between the numerically predicted (using FLAC) and the experimental stress-strain behaviour of three intact rocks are shown in Figs. 5 to 7 at different confining pressures tested. It can be observed that the predicted stress-strain curves closely follow the experimental data at all confining pressures for all the three intact rocks. Only for the case of Yamaguchi marble, slight deviation with the experimental results was observed (Fig. 7). But the amount of variation is very small considering the highly nonlinear behaviour of the particular rock. From these three figures, it is confirmed that the stress-strain response of intact rocks follows hyperbolic shape. The increase in the deviator stress with the increase in confining pressure, the effects of various properties of the intact rock, namely the shear strength properties, hyperbolic parameters are well represented in the numerical analysis.
4.2 Numerical Analysis of Jointed Rocks

The analyses of triaxial compression tests on jointed rocks were also performed using displacement control method discussed earlier. A *FISH* function was written in *FLAC* to determine the elastic modulus of the jointed rock with the regression equation developed from the present study. The *FISH* function obtains the value of modulus ratio \( E_r \) for the rock mass from Eq. 6 and then calculates the elastic modulus of the jointed rock by multiplying \( E_r \) with the elastic modulus of the intact rock \( (E_i) \). The *FISH* function is called at each iterative step in the numerical analysis. The jointed rocks selected for the analysis were Agra sandstone with single, two and three joints oriented at an angle of 70º (Arora, 1987) and block jointed gypsum plaster with two sets of joints oriented at 30º/60º (Brown and Trollope, 1970) were selected for the numerical study. The black jointed gypsum plaster is referred as gypsum plaster-2 in this paper to differentiate it from the other gypsum plaster referred as gypsum plaster-1 in Figs. 1 to 3, whose experimental data is used for the regression analysis. Among these tests, triaxial tests on Agra Sandstone were conducted on cylindrical samples of dimensions 38 mm diameter and 76 mm height whereas triaxial tests on gypsum plaster-2 were conducted on cubic samples of dimensions 100 mm length, 100 mm width and 200 mm height. The properties of the intact rocks and the properties of the joints used as input for this set of analyses are given in Table 5. Joint frequency reported in this table is calculated as number of joints per meter depth from the number of joints present in the triaxial samples. Table 5 also gives the confining pressures at which these jointed rocks were tested.

The results from the numerical analyses of triaxial tests on jointed rock soil samples and their comparison with the experimental data are shown in Figs. 8 to 10. Figure 8 gives the comparison of predicted stress-strain curves with experimental results for three cases of Agra sandstone with single, two and three joints inclined at an angle of 70º to the major principal stress direction respectively, tested at a confining pressure of 5 MPa. The numerical results are reasonably close to the experimental curves. This shows that the numerical model as discussed above is representative of the actual jointed rock sample. Thus it can be proclaimed that the effect of joint factor, which represents the number of joints in the jointed rock, is simulated well in the numerical model. Figure 9 shows the comparison of numerical and experimental results of triaxial tests on Agra sandstone samples with three joints inclined at 70º tested at three different confining pressures i.e. 2.5, 5 and 10 MPa. From the figure, it can be observed that the numerical model captured the stress-strain response of jointed rock at different confining pressures very well. Thus the validity of the model for jointed rocks with different joint factor tested at different confining pressures is confirmed from the present study.
Numerical analysis of block jointed gypsum plaster-2 with two sets of joints (30°/60°) exhibiting highly nonlinear stress-strain behaviour is next taken up. Analysis was carried out for four different confining pressures at which the

Table 5 - Properties of intact rock and joints used for the numerical analysis of jointed rocks

<table>
<thead>
<tr>
<th>Rock type</th>
<th>Properties of the intact rock</th>
<th>Properties of joints</th>
<th>Failure ratio (Rf)</th>
<th>Confining pressure (σ3) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hyperbolic properties</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Modulus number (K)</td>
<td>Exponent (n)</td>
<td>Cohesion (c)</td>
<td>Angle of internal friction (ϕ)</td>
</tr>
<tr>
<td>Agra sandstone</td>
<td>10196</td>
<td>0.14</td>
<td>19.22</td>
<td>51°</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gypsum Plaster-2</td>
<td>9000</td>
<td>0.7</td>
<td>0</td>
<td>34.5°</td>
</tr>
</tbody>
</table>
experiments were carried out. Results from this set of analyses are presented in Fig. 10. Figure 10 illustrates that the numerical model was successful in predicting the highly nonlinear behaviour of the block jointed sample. The matching between the actual results and predictions in this case is not as good as it was in other cases reported above. Considering the approximations involved in formulating the joint factor to represent the total behaviour of joint system, slight deviation observed in the present set of analyses on block jointed samples is reasonable.

Summarizing the results from all the sets of numerical analyses, it is clear that present numerical model can be successfully employed to simulate any type of jointed rock with any type of joint system tested at in triaxial compression any confining pressure. The validity of the regression models equations by Ramamurthy (1994) and Sitharam et al. (2001a) was verified for field cases of excavations with jointed rock masses by Varadarajan et al. (2001), Sitharam et al. (2001a) and Sitharam et al. (2001b). The regression equation developed in the present study is much simpler than the equations used in the analysis of above case studies. The applicability of the equation for the prediction of stress-strain behaviour of various jointed rock samples in triaxial compression is established from the present study. The same equation can also be used for simulating the field problems involving jointed rock masses, with less complexity than the other previously developed regression equations.

5. CONCLUSIONS

The constitutive behaviour of the jointed rock can be represented by a simple regression equation using equivalent continuum approach based on joint factor. This equation has been incorporated in the numerical program FLAC to obtain the elastic modulus of the jointed rock from the properties of the intact rock and the joint factor. Results from numerical simulation of triaxial compression tests on various intact and jointed rocks showed that the present equivalent continuum model works very well for different types of single, multiple and block jointed rocks with different joint fabric, joint orientation under a wide range of confining pressures. The numerical model developed in the present study is efficient in simulating the effects of number of joints, strength of joint, orientation of joint, type of rock and confining pressure. The input data required for the analysis is the properties of intact rock and the joint factor. Using the numerical model developed from this study, rock mass behavior can be reasonably estimated of in the absence of detailed experimental data. This numerical model can be employed for field problems also to obtain the stress distributions around excavations in jointed rock masses.

References


Fig. 1 - Experimental data of triaxial compression tests on jointed rock samples conducted at a confining pressure of 1 MPa used for the regression analysis
Fig. 2 - Experimental data of triaxial compression tests on jointed rock samples conducted at a confining pressure of 5 MPa used for the regression analysis
Fig. 3 - Experimental data of triaxial compression tests on jointed rock samples conducted at a confining pressure of 7 MPa used for the regression analysis
Fig. 4 - Grid used to simulate the triaxial test sample of rock in numerical analysis

Fig. 5 - Comparison of experimental and numerical stress-strain behaviour of intact sandstone (experimental data after Arora, 1987)
Fig. 6-Comparison of experimental and numerical stress-strain behaviour of intact Haizume siltstone (experimental data after Hoshino, 1972)
Fig. 7 - Comparison of experimental and numerical stress-strain behaviour of intact Yamaguchi marble (experimental data after Gokhale and Ramamurthy, 1981)
Fig. 8 - Comparison of experimental and numerical stress-strain behaviour of Agra sandstone with varying joint frequency (experimental data after Arora, 1987)
Fig. 9- Comparison of experimental and numerical stress-strain behaviour of Agra sandstone at different confining pressures (experimental data after Arora, 1987)
Fig. 10 - Comparison of experimental and numerical stress-strain behaviour of block jointed gypsum plaster-2 at different confining pressures (experimental data after Brown and Trollope, 1970)