Underground Excavation in Jointed Medium

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ABSTRACT

An existing continuum characterization model of a jointed medium is implemented in a computer program for nonlinear finite infinite element analysis. The implementation is verified by analyzing a deep circular opening subject to a hydrostatic initial stress field and different joint orientations with strike parallel to the tunnel axis. This software is then used in the study of the effect of joint orientation on the location and extent of the plastic zone around deep circular opening subject to a non-hydrostatic initial stress field. A shallow circular opening is subjected to an internal pressure and the effect of different joint orientation is also studied. The no-tension zones around a deep powerhouse cavern are also shown to depend upon the joint orientation. The continuum characterization model offers a simple means to analyze the underground openings more realistically.

Key words: *Anisotropy; Equivalent continuum; Finite Element Analysis; Infinite Element; Jointed medium; Tunnels; Underground openings*

INTRODUCTION

The underground space is extensively utilized in hydroelectric power generation projects, metro tunnels and storage of a large number of essential commodities. For a safe and successful completion of the project, it is necessary to foresee the response of underground medium to excavation. The present state of the art shows that the analytical predictions do not satisfactorily agree with the actual field measurements in a number of case histories. The empirical methods, which have been derived from field experience and actual case histories, usually do a better job. This inadequacy of the analytical methods may be attributed to the simplifications, which have to be introduced, to limit the problem size and also to bring the problem with in the scope of the method of analysis. One such simplification is to assume the medium of excavation as isotropic and homogeneous. The actual underground medium is complex due to the presence of discontinuities like joints, cracks, faults and bedding planes. Although, an exact analytical description of such a medium appears difficult, it may be possible to bring the analysis closer to reality by using an equivalent continuum description.

This paper presents analysis of excavation in such an equivalent medium. Only a single set of joint with strike parallel to the tunnel axis is considered. A deep circular opening and a deep powerhouse cavern subjected to a non-hydrostatic initial stress field and a shallow circular opening subjected to an internal pressure are analyzed. The orientation of joints in the medium is varied and the resulting medium is treated as a continuum so that the Finite Element Method (FEM) is applicable. Also, the far field of the problem is modeled by mapped infinite elements so that the analysis is closer to the Boundary Element Method (BEM). This study shows that the extent and location of plastic and no-tension zones are sensitive to the joint orientation.

MODELING OF JOINTED MEDIUM

State of the Art

The strength of jointed rock mass was studied through model tests with variable joint pattern by Brown (1970). Two different analytical approaches have been used to deal with this subject. The Distinct Element Method (DEM) pioneered by Cundall (1971) uses a dis-continuum description. This method and a multi-laminate strategy for jointed medium are described by Pande et al.(1990). The numerical models for rock discontinuity were proposed by Goodman et al. (1968), and Heuze and Barbour (1982). These can be used to model individual discontinuities. However, such an analytical model is likely to become cumbersome if the number of discontinuities is large.

The equivalent continuum approach is an alternative to DEM and modeling of individual joints through joint elements. The jointed medium is replaced by a continuum, the properties of which depend upon the properties of the intact medium and joint set characteristics. One such continuum model was proposed by Morland (1974). It was subsequently elaborated by Chen (1989, 1990). Another continuum model of a jointed medium was developed by Singh (1973). Although, Singh (1973) demonstrated its utility in the elastic analysis of jointed medium, its application in the nonlinear applications has not been attempted. This is achieved in the present study.

The DEM is very useful in the study of jointed medium, however, a continuum model has the advantage in that it fits in the FEM framework so that the analysis can be performed without requiring any specialized treatment. These two approaches were compared by Fairhurst and Pai (1990) with respect to the problems of underground excavation. It was found that in the normal operational range the results of these two methods are comparable. Any improvement in the FEM analysis may bring it closer to the existing specialized methods of analysis. For example, in the present study the near and far fields of the analysis domain are represented by finite and infinite elements, respectively. It is possible to account for a non-zero far field decay of initial stress without having to create nodes at infinity (Kumar, 1999). This representation narrows the gap between FEM and BEM. This feature is not available in the Discrete Element Analysis and it still uses a truncation approach to deal with the unbounded analysis domains. A coupling between DEM and FEM is described by Dowding et al. (1983) in which the infinite elements are not considered.

Description of Continuum Model

Some important relations of the continuum model, which is used in this study, are as given below. The full details may be found in the original publication by Singh (1973). To simplify the model, it is assumed that the joints in the set are parallel and continuous. The strike of joints is parallel to the tunnel axis. Let the joint frequency be n, therefore joint spacing s = 1/n. The compliance matrix of the jointed rock mass [M] is written as,

$$[M] = [M_r] + n\,[K_j]^{-1} \tag{1}$$

where, $[M_r]$ = compliance matrix of the intact rock; and

$$[K_j]^{-1} = \begin{bmatrix} \frac{0}{-} & 0 & 0 \\ 0 & \frac{1}{K_N} & 0 \\ 0 & 0 & \frac{1}{K_T} \end{bmatrix}$$

K_N = Joint normal stiffness, and
K_T = Joint shear stiffness.

In an expanded form, the Eq. 1 for a two dimensional plane strain formulation is written as,

It is seen in Eq. 2 that the normal compliance coefficient is affected when a joint set perpendicular to it exists. The Eqs. 1 and 2 for the case of two mutually orthogonal and continuous joint sets may be written as Eq. 3.

$$[\mathrm{M}]=\begin{bmatrix} \dfrac{1-\upsilon_r^2}{E_r} & \dfrac{-\upsilon_r(1+\upsilon_r)}{E_r} & 0 \\ \dfrac{-\upsilon_r(1+\upsilon_r)}{E_r} & \dfrac{1-\upsilon_r^2}{E_r}+\dfrac{n}{K_N} & 0 \\ 0 & 0 & \dfrac{2(1+\upsilon_r)}{E_r}+\dfrac{n}{K_T} \end{bmatrix} \qquad (2)$$

$$[\mathrm{M}] = [\mathrm{M_r}] + \mathrm{n_1}\,[\,\mathrm{K1_j}\,]^{-1} + \mathrm{n_2}\,[\,\mathrm{K2_j}\,]^{-1} \qquad (3)$$

where,

$$[K1_j]^{-1}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \dfrac{1}{K1_N} & 0 \\ 0 & 0 & \dfrac{1}{K1_T} \end{bmatrix} \quad \text{and} \quad [K2_j]^{-1}=\begin{bmatrix} \dfrac{1}{K2_N} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{K2_T} \end{bmatrix}$$

The subscripts 1 and 2 refer to joint set 1 and 2, respectively with frequency n_1 and n_2, respectively. The Eq. 3 in an expanded form is written as,

$$[M]=\begin{bmatrix} \dfrac{1-\upsilon_r^2}{E_r}+\dfrac{n_2}{K2_N} & \dfrac{-\upsilon_r(1+\upsilon_r)}{E_r} & 0 \\ \dfrac{-\upsilon_r(1+\upsilon_r)}{E_r} & \dfrac{1-\upsilon_r^2}{E_r}+\dfrac{n_1}{K1_N} & 0 \\ 0 & 0 & \dfrac{2(1+\upsilon_r)}{E_r}+\dfrac{n_1}{K1_T}+\dfrac{n_2}{K2_T} \end{bmatrix} \qquad (4)$$

It is seen that the equivalent continuum is anisotropic in which the principal strength directions may or may not coincide with the principal directions of the structure. The effect of a single joint set may be affected by modifying the values of modulus of elasticity and shear modulus as follows,

$$E_r \rightarrow \left[\frac{E_r}{1 + n\left(\frac{E_r}{K_N} \right)} \right] \quad \text{and} \quad G_r \rightarrow \left[\frac{G_r}{1 + n\left(\frac{G_r}{K_T} \right)} \right] \tag{5}$$

where $G_r = \dfrac{E_r}{2.(1 + \upsilon_r)}$

If there are two mutually perpendicular joint sets, then, the shear modulus is modified on account of both. Thus, it is the shear modulus, which suffers most on account of joints. The compliance matrix is rotated through standard matrix operations if the joint sets do not coincide with the principal structural axes.

Implemetation

This equivalent continuum model is implemented in a nonlinear FEM analysis computer program and the results presented in the subsequent sections are obtained. Only a single set of joints is considered.

DEEP CIRCULAR OPENING

In this application, a deep circular opening located in an initially stressed medium is analyzed for the plastic region. The analysis under hydrostatic initial stress field is used for validation while non-hydrostatic initial stress field is employed for more realistic solutions.

Problem Geometry

Radius of opening	= 200 cm
Depth of opening from free surface	= 500 M

Medium Description

Modulus of elasticity E	= 34500.0 Kg/cm^2
Poisson Ratio	= 0.2
Medium density	= 2300 Kg/m^3
Cohesion	= 30.0 Kg/cm^2
Angle of internal friction	= 30^0
Yield law	= Mohr-Coulomb
Flow rule	= Associated

Joint Set Properties

Joint spacing	= 20 cm
Joint normal stiffness	= 0.1 E_r
Joint shear stiffness	= 0.01 G_r
Joint inclination	= Variable between 0° and 90°

Loading

Strength of vertical initial stress	= 115 Kg/cm^2
Initial stress ratio (horizontal / vertical)	= 2.0

Numerical Model

The finite-infinite mesh of the problem is shown in Fig. 1. It contains 180 nodes. There are 48 finite elements (8-node isoparametric quadrilateral) and 6 infinite elements (5-node mapped with an inverse type far field decay characteristics). These elements model the near and the far fields of the problem, respectively. By virtue of a four-fold symmetry only a quarter of the problem is analyzed in the plane strain formulation of the theory of elasticity. Since the medium containing the opening is going to be jointed in which case the four-fold symmetry will be lost. However, the jointed medium is converted to an equivalent continuum, which restores the symmetry. Therefore, the same mesh is applied in the study.

Validation (Hydrostatic initial stress)

The first analysis is performed for an initial stress ratio of 1.0. The plastic region around the opening in an isotropic medium is shown in Fig. 2a. The numerical results for joint orientations of 0° and 60° are shown in Figs. 2b and 2c, respectively. Since the problem of jointed medium under hydrostatic initial stress field is essentially one-dimensional, a change in joint orientation should merely rotate the plastic region through the angle of joint inclination. This feature is easily available in Figs. 2b and 2c.

Numerical Results (Non-Hydrostatic initial stress)

The numerical results of isotropic and jointed media under non-hydrostatic initial stress (initial stress ratio 1.5) are shown in Figs. 3a to 3h. The joint orientation is varied between 0° and 90° with an increment of 15°.

Discussion

Figs. 3a to 3h show that the joint orientation is an important factor in determination of the extent of plastic region around the openings. A section from the rock mass classification of Bieniawski (1984) is reproduced in Table 1. For the strike of joints parallel to the tunnel axis, it gives the joint orientations between 45° and 90° as very unfavorable. The basis on which Table 1 has been derived is not readily available in the published literature. The results of Fig. 3 show that the extent of plastic region is largest for joint orientation of around 60° and smallest for joint orientation of 30°. This is in line with the recommendations of Bieniawski (1984) contained in Table 1. This shows that the equivalent continuum model is capable of simulation of field experience as well as its quantification. Kaiser (1986) has also cited a Japanese study to show the influence of joint orientation on the plastic region.

Table 1 - Effect of joint orientations in tunnelling (Bieniawski, 1984)

Strike perpendicular to the tunnel axis				Strike parallel to the tunnel axis		Dip angle 0° to 20° irrespective of strike
Drive with Dip		Drive against Dip				
Dip 45°-90°	Dip 20°-45°	Dip 45°-90°	Dip 20°-45°	Dip 45°-90°	Dip 20°-45°	
Very favorable	Favorable	Fair	Un-favorable	Very un-favorable	Fair	Unfavorable

SHALLOW CIRCULAR OPENING

A shallow circular opening located in an isotropic or jointed medium is subjected to internal water pressure.

Problem Description

Depth of center of opening from free surface	= 750.0 cm
Radius of opening	= 200 cm

Medium Properties

Modulus of elasticity	= 34500.0 Kg/cm^2
Poisson Ratio	= 0.20
Medium density	= 2400.0 Kg/cm^2
Cohesion	= 20.0 Kg/cm^2
Angle of internal friction	= 30°
Yield law	= Mohr Coulomb
Flow rule	= Associated

The joint set properties are as given previously.

Numerical Model

The numerical model of this problem is shown in Fig. 4. It contains 226 nodes. The near and the far fields of the problem are represented by 58 finite elements and 12 infinite elements. By virtue of symmetry, only half of the problem is analyzed in plane strain formulation.

Numerical Results

The numerical result for an isotropic medium is given in Fig. 5a. Figures 5b to 5e show numerical results for the jointed medium with joint orientation varying from 0° to 90° with an increment of 30°.

Discussion

The actual problem involving water pressure and jointed medium is coupled. The joints may open under the action of water pressure and thus may alter the distribution of water pressure. Such problems have been discussed by Barton et al, 1985; Barton, 1986 and Li and Brown, 1988. The equivalent continuum approach such as the one used in this study does not allow this hydro-mechanical coupled analysis which involves simulation of opening and closing of joints. It stops after giving the region of influence. It can, however, be seen in Fig. 5 that the results obtained by the use of continuum model are sensitive to the joint orientation.

UNDERGROUND POWERHOUSE CAVERN

A no-tension analysis is performed to establish feasibility of a structure founded in or on rock mass. The strategy for such an analysis was originally proposed by Zienkiewicz et al. (1968). This strategy has now been refined by Kumar, 1999. In the refined analysis, the location and extent of the no-tension region depends upon the tensile strength properties of the medium and the formulation is in the standard format of an elasto-visco-plastic nonlinear finite element analysis. The available solutions are valid for an isotropic medium. In this section, the problem of an underground powerhouse cavern is solved for no-tension zone and the influence of joint orientation on it is studied.

Problem Data

The structural configuration is shown in Fig. 6. The opening center is located at a distance of 112 meters from the free surface. This opening is treated as deep.

Medium Description

Modulus of elasticity E	= 1410000 Tf/m^2
Poisson ratio ν	= 0.15
Unit weight of material γ	= 2.5 Tf/m^3
Tensile strength of medium	= 15 Tf/m^2
Initial stress ratio	= 0.2

Loading

The deep opening is subjected to a uniform initial stress corresponding to the weight of the overburden material at the center of the opening.

Intensity of vertical initial stress	= 280 Tf/m^2
Intensity of horizontal initial stress	= 56 Tf/m^2

Numerical Model

The numerical model of this problem shown in Fig. 7 contains 462 nodes. The near and far fields of the problem are represented by 130 finite and 24 infinite elements, respectively. This problem does not possess any symmetry and is analyzed in a plane strain formulation.

Numerical Results

The numerical results are shown in Fig. 8. For the purpose of comparison, the first analysis is for an isotropic medium. This result is similar to that reported by Zienkiewicz et al (1968). The remaining results of Fig. 8 show that the no-tension region is also sensitive to the joint orientation. It varies from virtually none (Fig. 8h) to a maximum at the top and bottom in Figs. 8e and 8f.

CONCLUDING REMARKS

The underground medium usually contains discontinuities because of which the medium can not be described as isotropic. Yet this medium description is often used in the analysis of underground structures. In this paper, a published equivalent continuum model of a jointed medium is implemented in a computer program for nonlinear finite element analysis. This model and its implementation are verified by solution of a deep circular opening in a hydrostatic initial stress field. Subsequently, this software is used to study influence of joint orientation on the plastic and no-tension regions under non-hydrostatic initial stress and internal water pressure. The analysis shows that the joint orientation may drastically alter the location and extent of plastic and no-tension regions. Some guideline to this effect is available in the RMR rock mass classification system, which supports the results of present study (Fig. 9). This equivalent continuum model presents a simple means for a more realistic treatment of the problem of underground structures.

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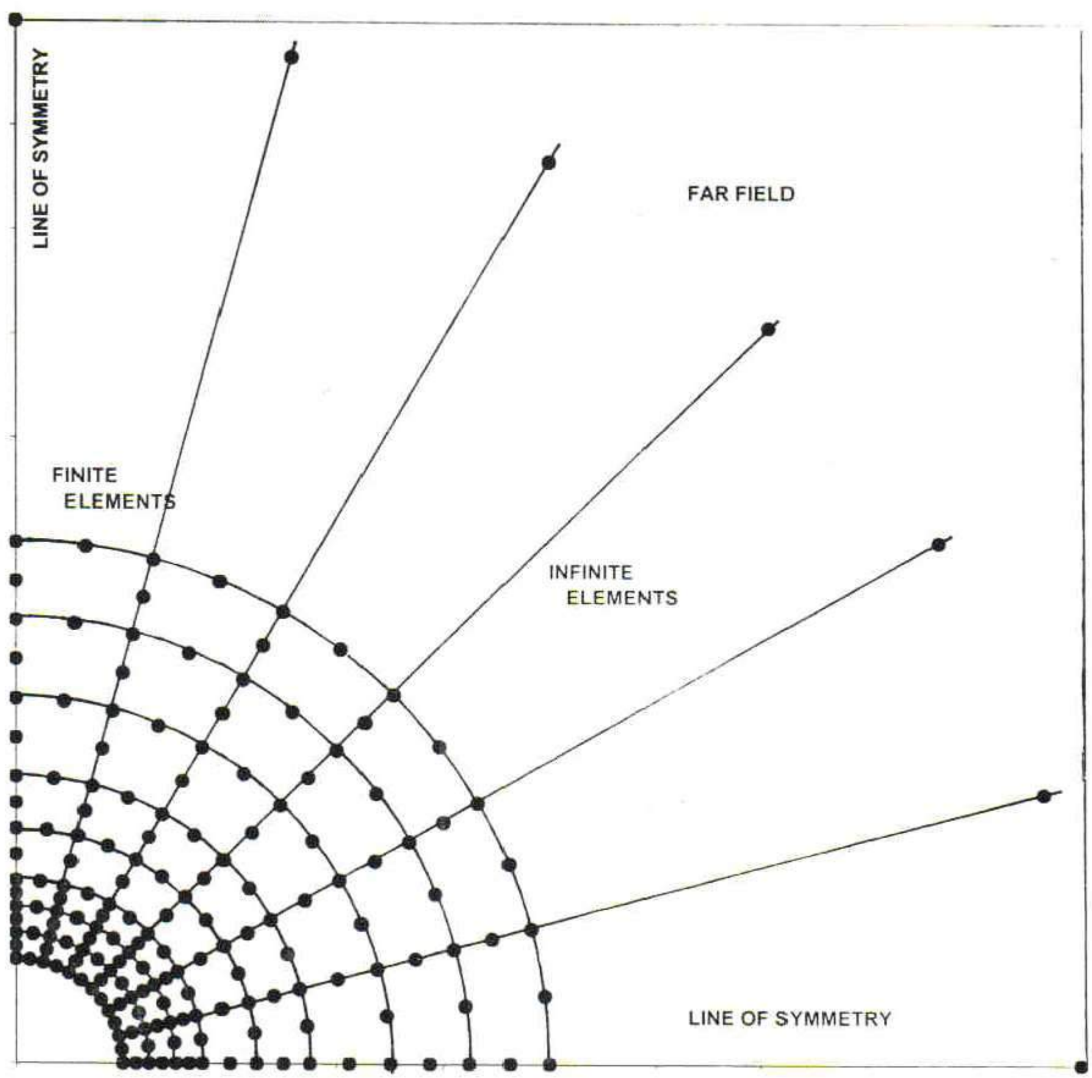
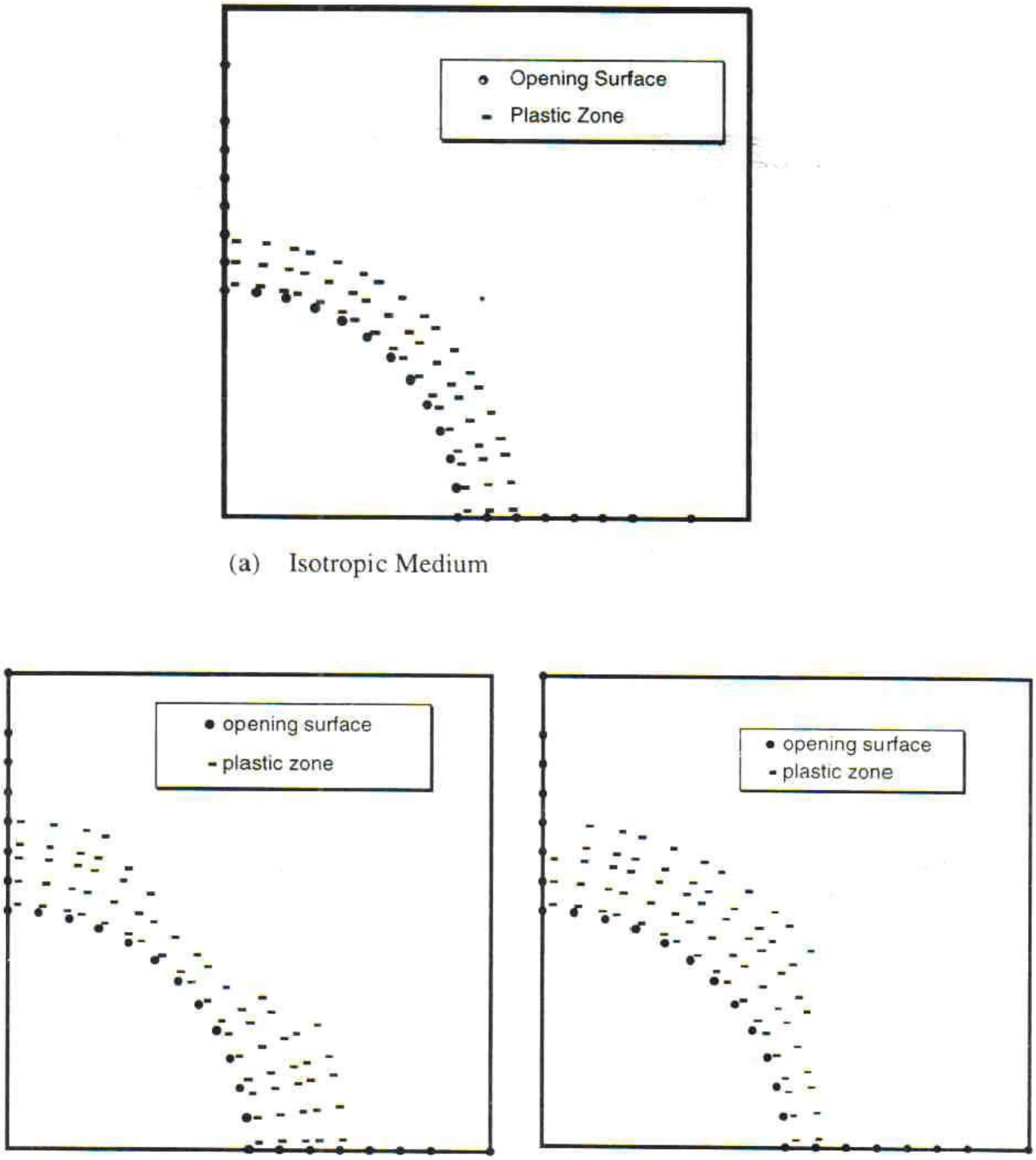


Fig. 1- Numerical model of a deep circular opening



(a) Isotropic Medium

(b) Joint orientation 0°

(c) Joint orientation 60°

Fig. 2 - Plastic zone around deep circular opening under hydrostatic initial stress

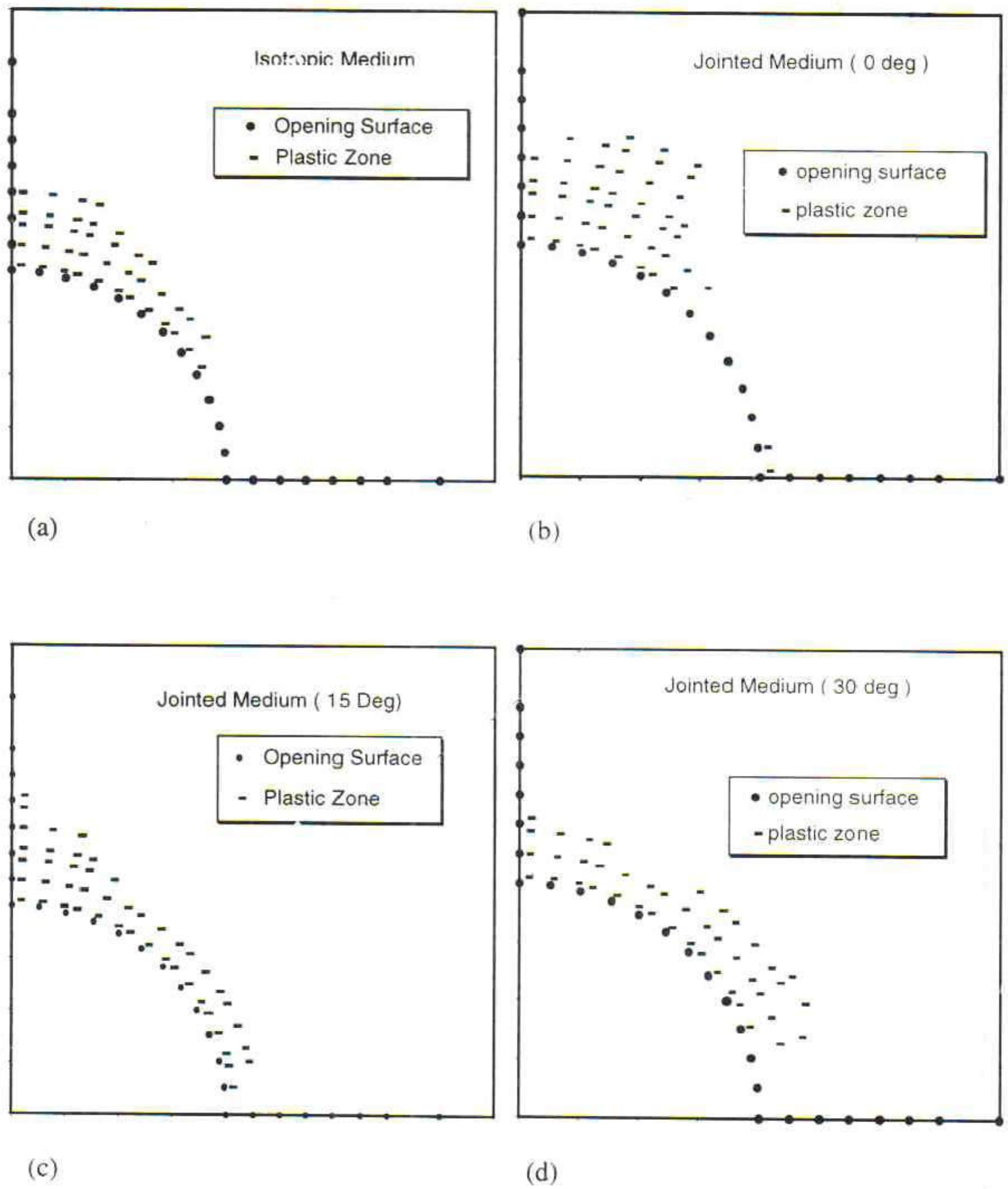


Fig.3 (a to d) - Plastic zone around deep circular opening under hydro-static initial stress

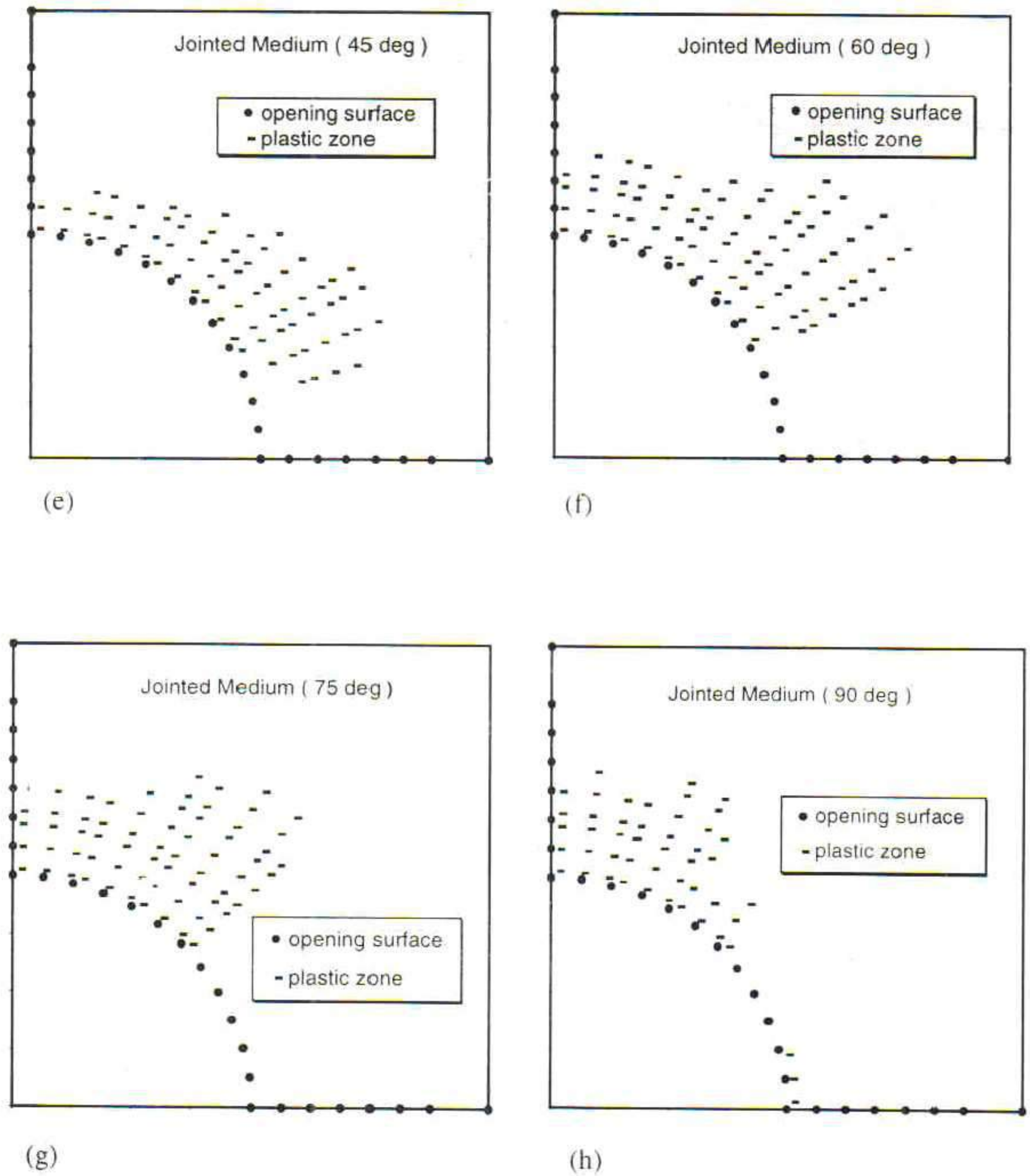


Fig. 3 (e to h) - Plastic zone around deep circular opening under hydrostatic initial stress

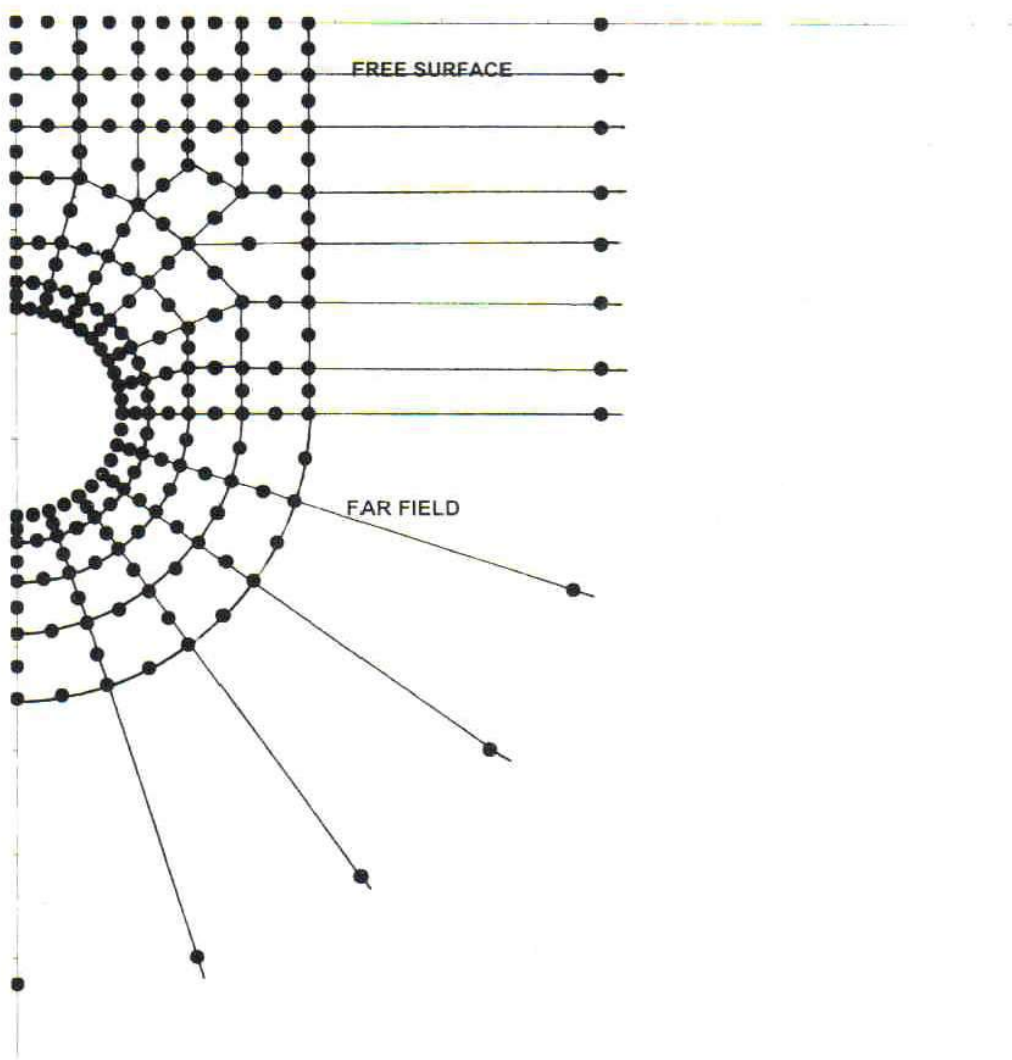


Fig. 4- Numerical model of a shallow circular opening

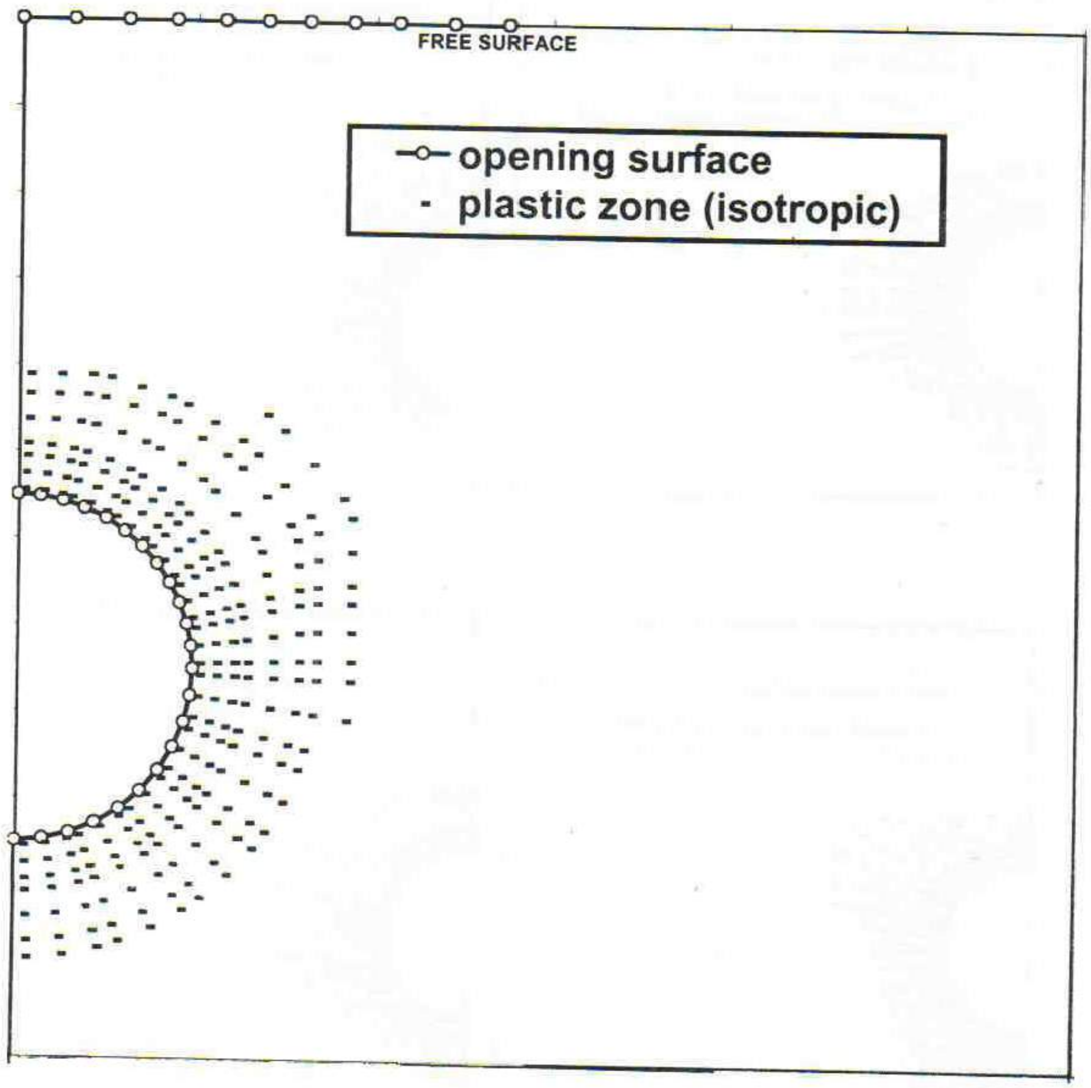


Fig. 5 a - Plastic zone due to internal pressure - isotropic medium

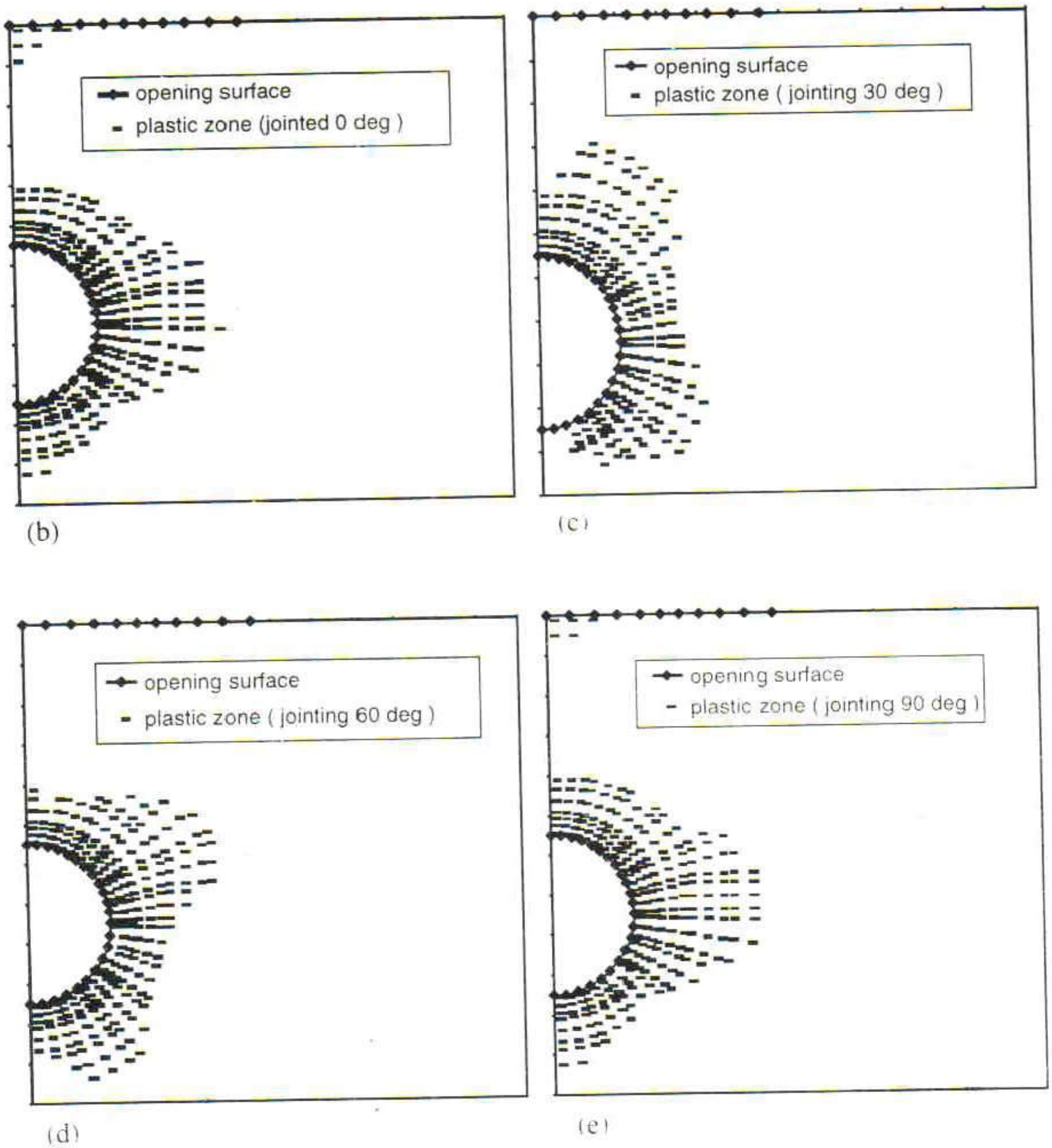


Fig. 5 (b to e) - Plastic zone due to internal pressure

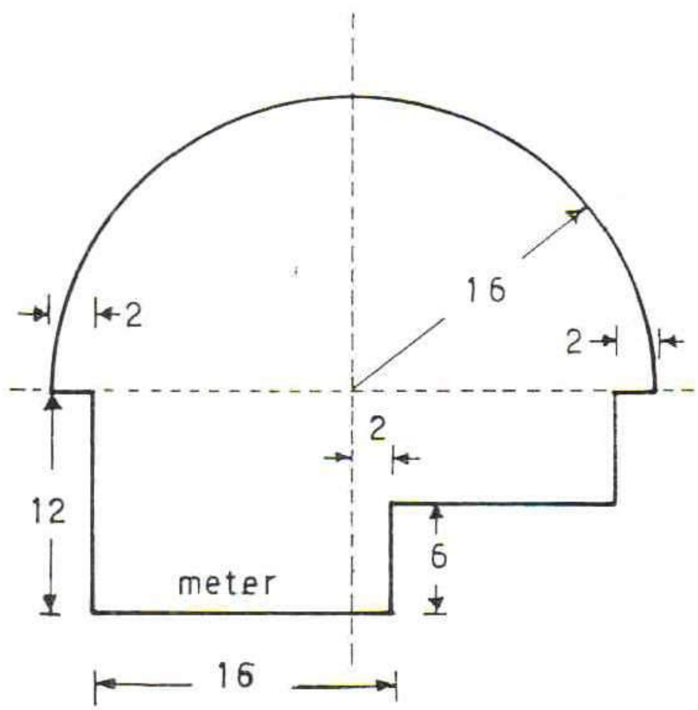


Fig. 6 - Geometry of a deep powerhouse cavern

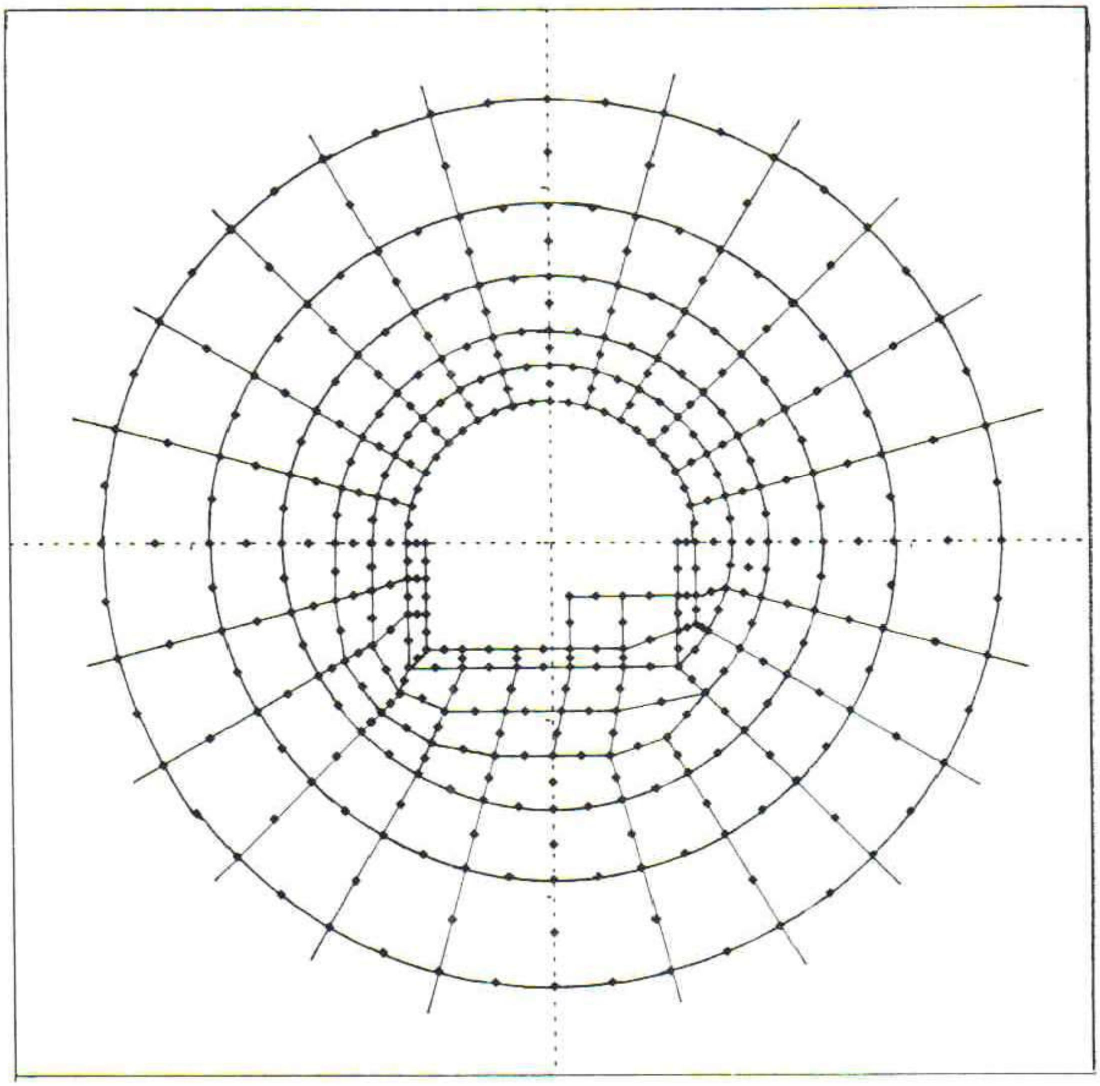
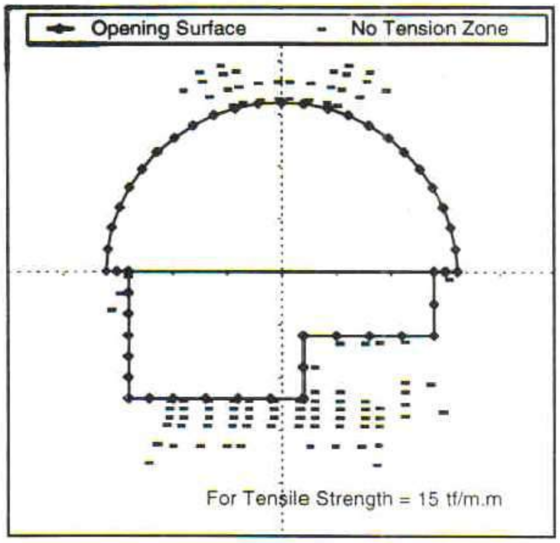
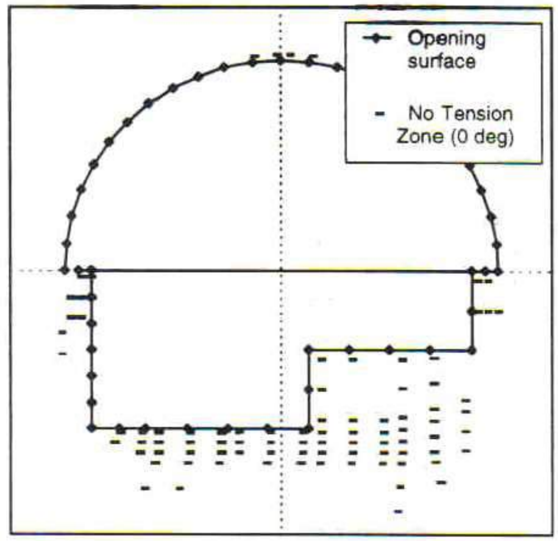


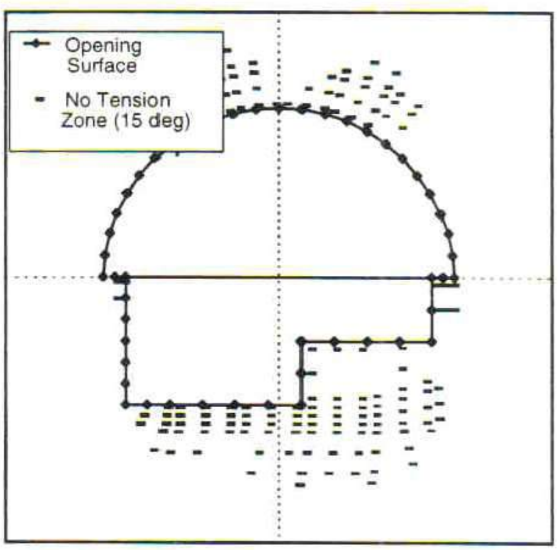
Fig. 7 - Numerical model of the deep powerhouse cavern



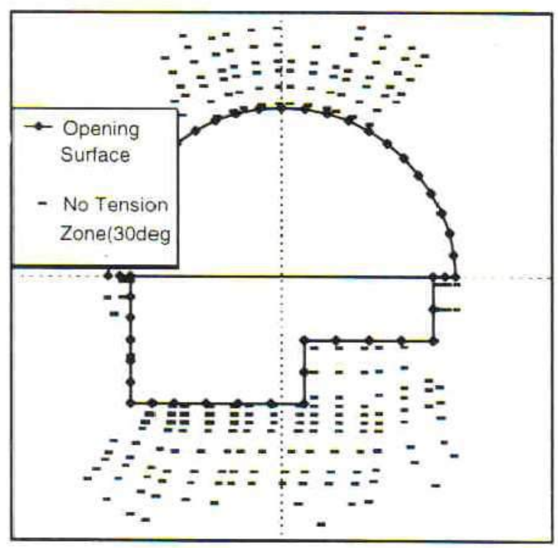
(a) Isotropic medium



(b) Joint Orientation 0°

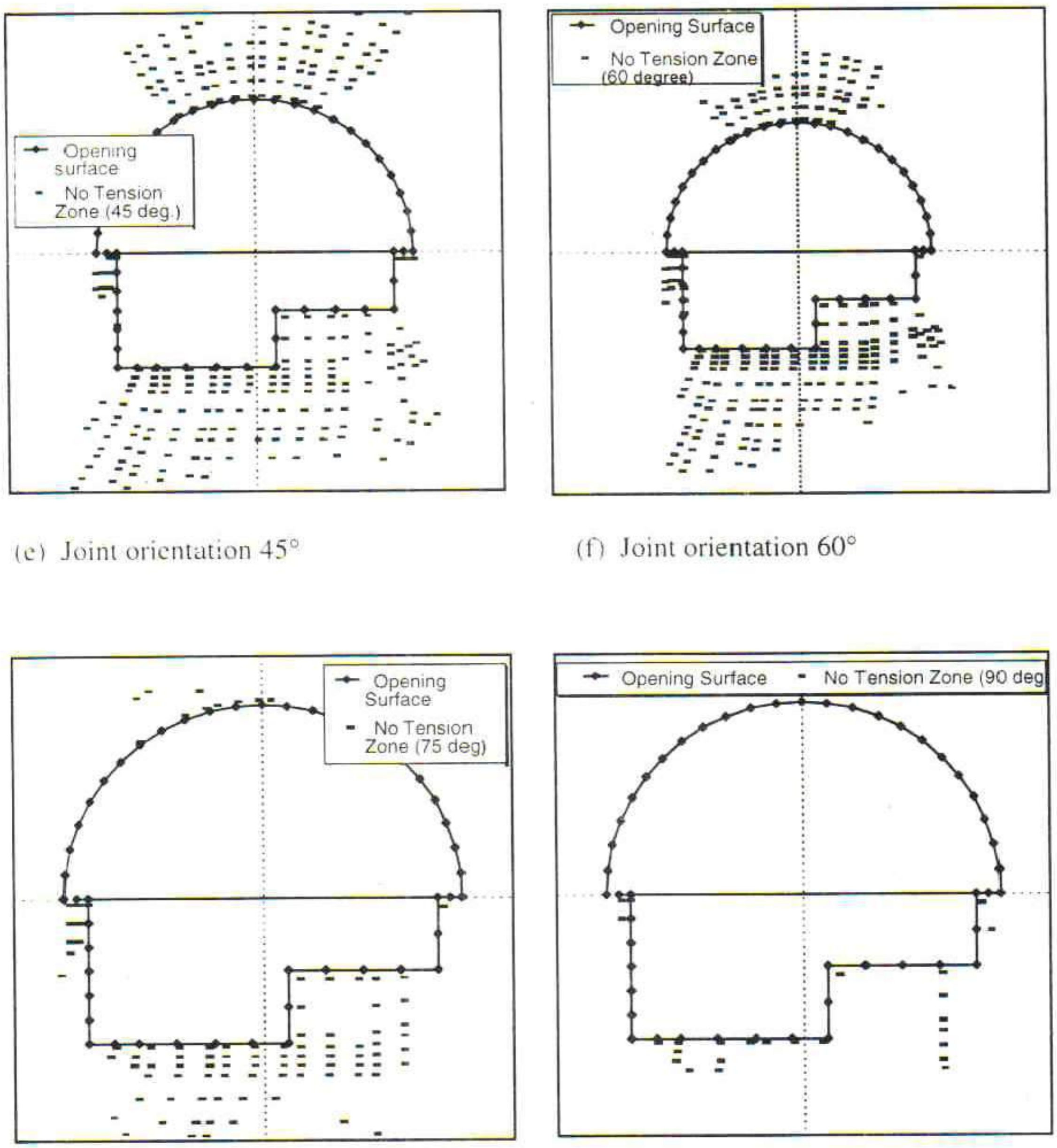


(c) Joint Orientation 15°



(d) Joint Orientation 30°

Fig. 8 (a to d)- No-tension zone around deep powerhouse cavern



(e) Joint orientation 45°

(f) Joint orientation 60°

(g) Joint orientation 75°

(h) Joint orientation 90°

Fig. 8 (e to h)- No-tension zone around deep powerhouse cavern

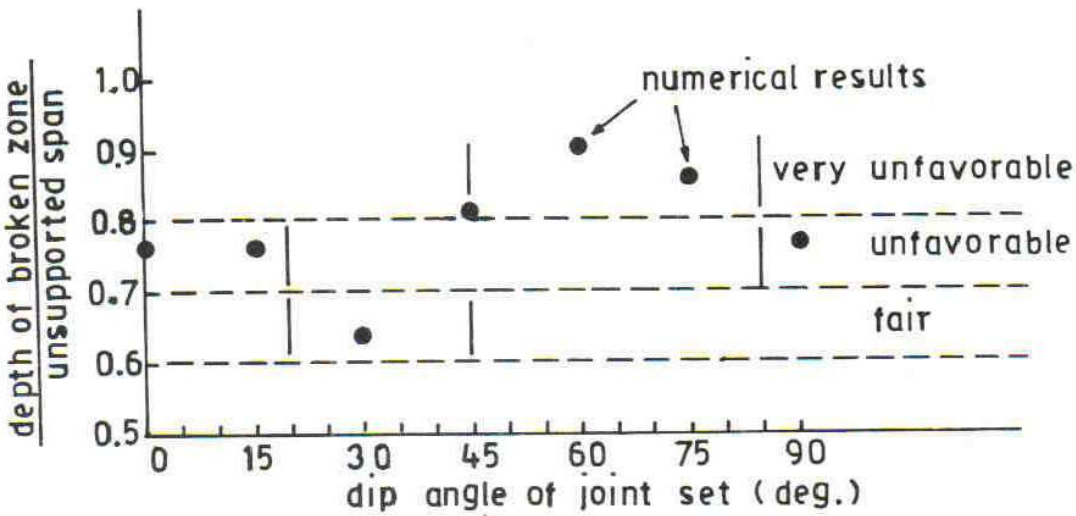


Fig. 9 - Comparison of RMR system with the numerical results of Fig. 3